GE2215 Lecture 11
Spatial Statistical Analysis

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Recap: Local operations

- Local operations are cell-by-cell operations
- Local operations can be operated on single or multiple rasters
  - Local operations on a single raster
    - Reclassification: one-to-one mapping and one-to-many mapping
  - Local operations with multiple rasters
    - Mathematical operations: $+$, $-$, $\times$, $\div$
    - Summary statistics: maximum, minimum, range, sum, mean, median, and standard deviation, which apply to numeric data only
    - Tabulation of measures: majority, minority, combine and number of unique values, which apply to both numeric and categorical data

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Recap: Focal operations

- A focal operation, also called a neighbourhood operation, involves a focal cell and a set of its surrounding cells.
- A neighbourhood is defined by shape and size
- Focal operations are usually performed on a single layer
  - Summary statistics: maximum, minimum, range, sum, mean, median, and standard deviation
  - Tabulation of measures: majority, minority, and variety

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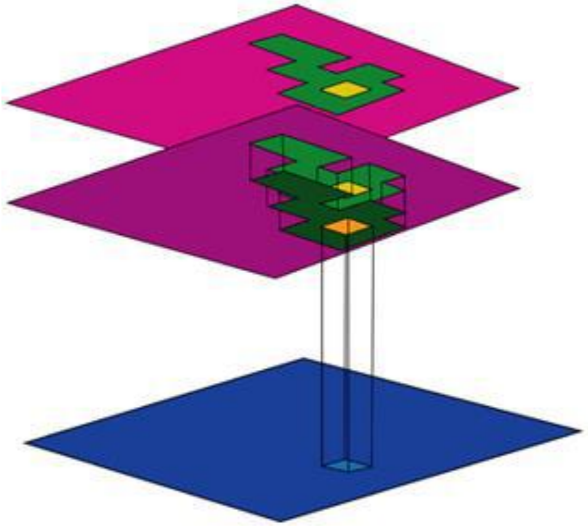
Recap: Zonal operations

- A zonal operation works with groups of cells of same values or like features.

  These groups are called zones

- Zones may be contiguous or non-contiguous



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Recap: Global operations

- Global operations are similar to zonal operations whereby the entire raster dataset's extent represent a single zone
- Global operations apply a bulk change to all cells in a raster

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Recap: Global operations

- Euclidean distance: calculating the closest distance away from a source
- Allocation: The cell value in an allocation raster corresponds to the closest source cell for the cell
- Direction: The cell value in a direction raster corresponds to the direction in degrees that the cell is from the closest source cell

1.0	**2**	1.0	2.0
1.4	1.0	1.4	2.2
1.0	1.4	2.2	2.8
1	1.0	2.0	3.0

2	**2**	2	2
2	2	2	2
1	1	1	2
1	1	1	1

90	**2**	270	270
45	360	315	287
180	225	243	315
1	270	270	270

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Recap: Raster data clip

- There are two methods to clip a raster
  - Specify an analysis mask (clip by mask)
  - Specify the minimum and maximum x-, y-coordinates (clip by rectangle)




(d) A rectangle

(a) Input raster (b) An analysis mask (c) Clip-by-mask result (e) Clip-by-rectangle result

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Recaps: Map algebra

- Map algebra basically involves doing math with maps
- The key difference with Algebra is that it only applies to rasters
- Map algebra can be local, focal, zonal and global
- Commonly used mathematical functions
  - Arithmetic operations
  - Statistical operations
  - Relational operations
  - Trigonometric operations
  - Exponential and logarithmic operations

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Outline

- Basics of Statistics
  - Descriptive Standard Statistics
  - Inferential Standard Statistics
- Introducing Spatial Statistics
  - Descriptive – Centrographic Statistics
  - Inferential – Point Pattern Analysis

    – Spatial Autocorrelation

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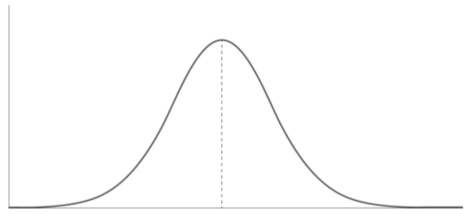
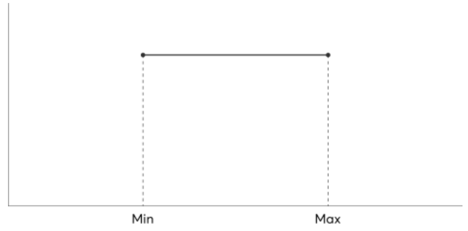
Standard Statistical Analysis

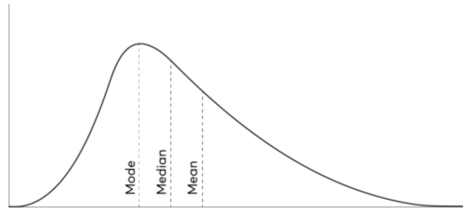
- Two parts of Standard Statistics (A quick view)
  1. **Descriptive statistics**
     - Obtain <u>summary</u> measures to <u>describe</u> a set of data
     - Three types of measures:
       - **<u>Central tendency</u>**: mean, median values
       - **<u>Variability</u>**: standard deviation
       - **<u>Frequency distributions</u>**



Normal Distribution



Uniform Distribution



Skewed Distribution

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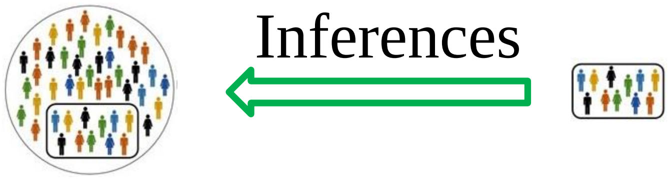
Standard Statistical Analysis

- Two parts of Standard Statistics (A quick review)

  2. Inferential statistics

  - Make inferences from a sample about a population
  - Some definitions
    - **Population**: all occurrences
    - **Sample**: a subset of the population
    - **Parameters**
    - **Statistics**



Population

- ***Parameters***
  - Values from the population

Sample

- ***Statistics***
  - Values from the sample

Statistics are estimates of parameters

An example: Are left-handed people more intelligent than right-handed people?

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    – Spatial Autocorrelation

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What Is Spatial Statistics

- Definition of Spatial Statistics
  - The field of study concerning statistical methods that use **space** and **spatial data**
  - Space and location matters

- **Spatial data**
  - Point: bus stops, grocery stores
  - Polyline: transportation networks
  - Polygon: parks, campus
  - Continuous data: temperature, precipitation

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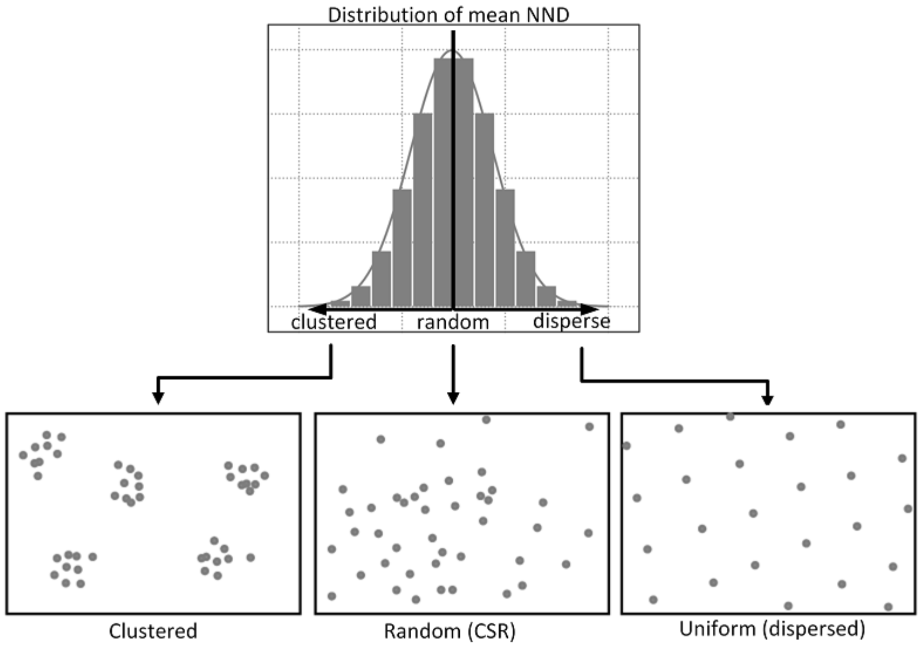
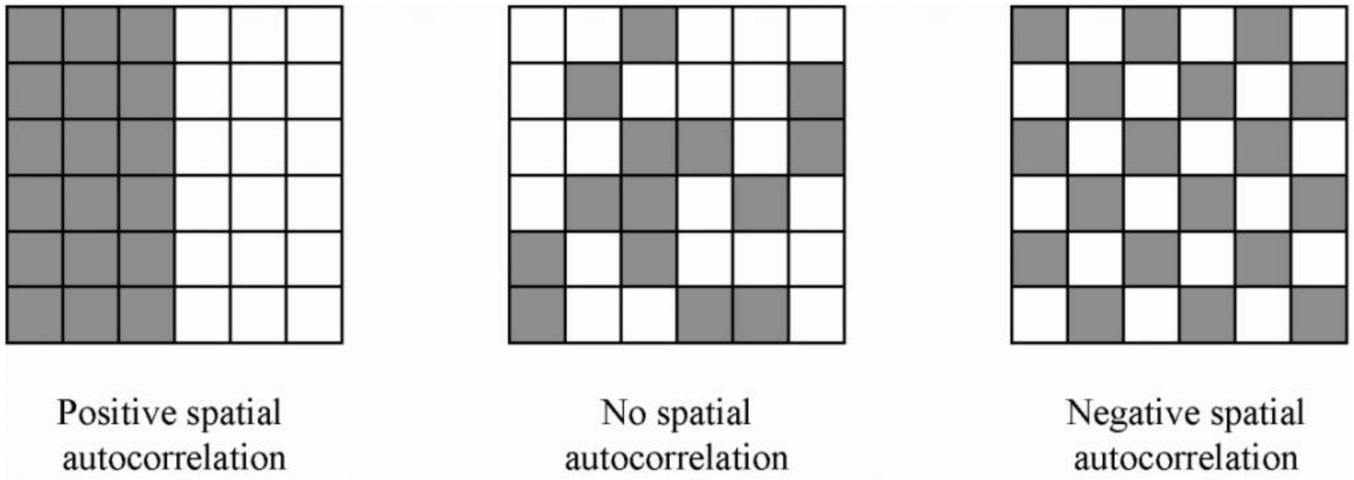
What Is Spatial Statistics

- **Similarly, spatial statistics also have two parts:**
  - **Descriptive** spatial statistics: Centrographic statistics
  - **Inferential** spatial statistics: e.g., Point Pattern Analysis, Spatial Autocorrelation





Nearby cases are similar

Neighbor cases have no particular relationship

Nearby cases are dissimilar

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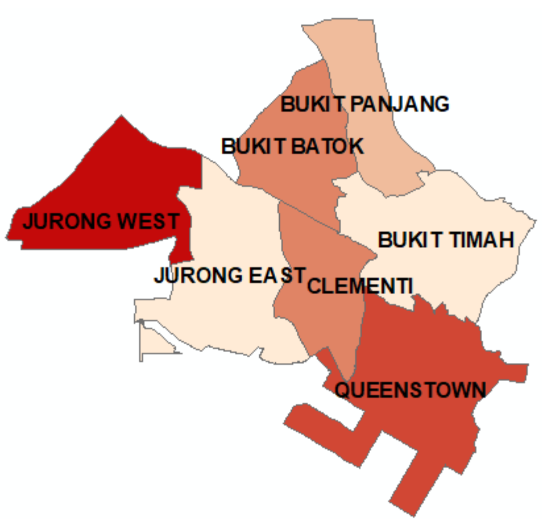
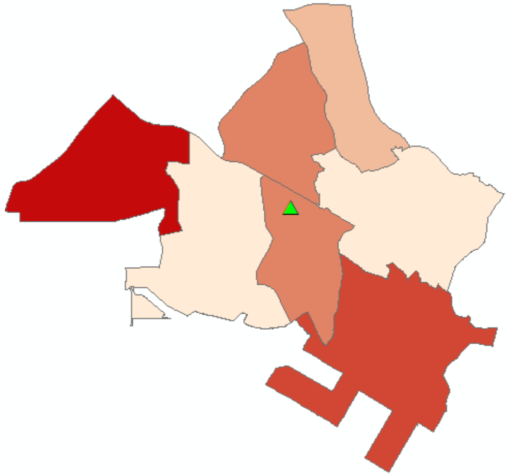
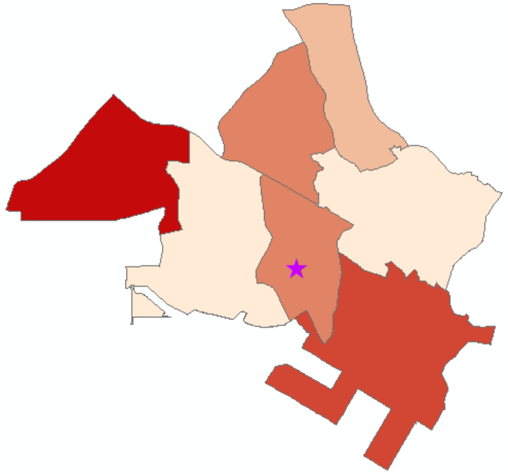
Descriptive Spatial Statistics: Centrographic Statistics

- Measures of Centrality
  - Centroid (Polygon data)
  - Weighted mean center





Geometric Center (Centroid)



Weighted mean center
(elder population center)

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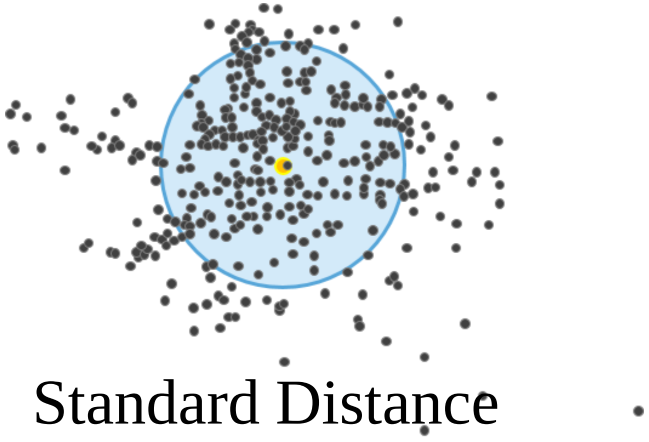
Descriptive Spatial Statistics: Centrographic Statistics

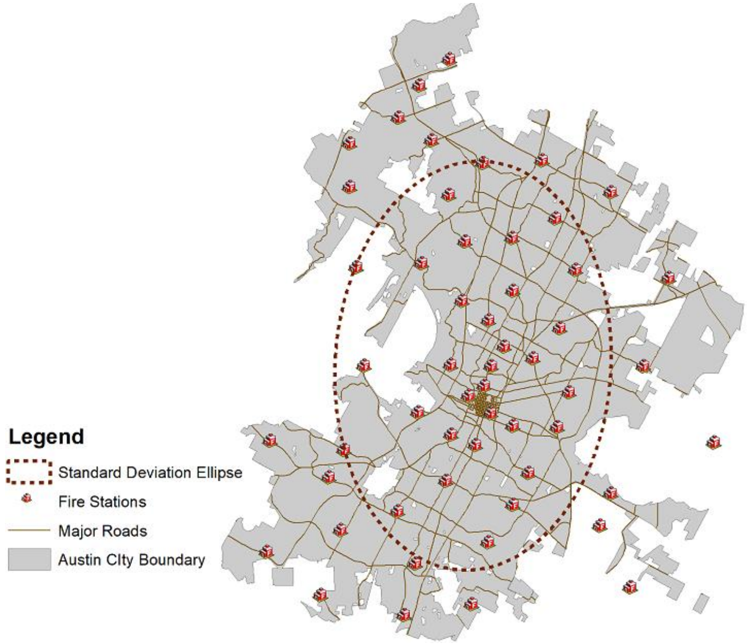
- Measures of Variance
  - Standard Distance
    - Measure the <u>degree</u> to which features are concentrated or dispersed around the geometric mean center
    - Does not capture the <u>shape</u> of distribution

  - Standard Deviational Ellipse (SDE)
    - Describe the <u>direction</u> of the dispersion
    - A typical method for measuring the activity space

A perfect circle here.
Size of circle depends on how well distributed the points are



Standard Distance



Standard Deviational Ellipse

https://gistbok.ucgis.org/bok-topics/point-pattern-analysis

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- Review (Refreshing your memory) of Statistics
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- Introducing Spatial Statistics
  - Descriptive – Centrographic Statistics
  - **Inferential – Point Pattern Analysis**

    – Spatial Autocorrelation

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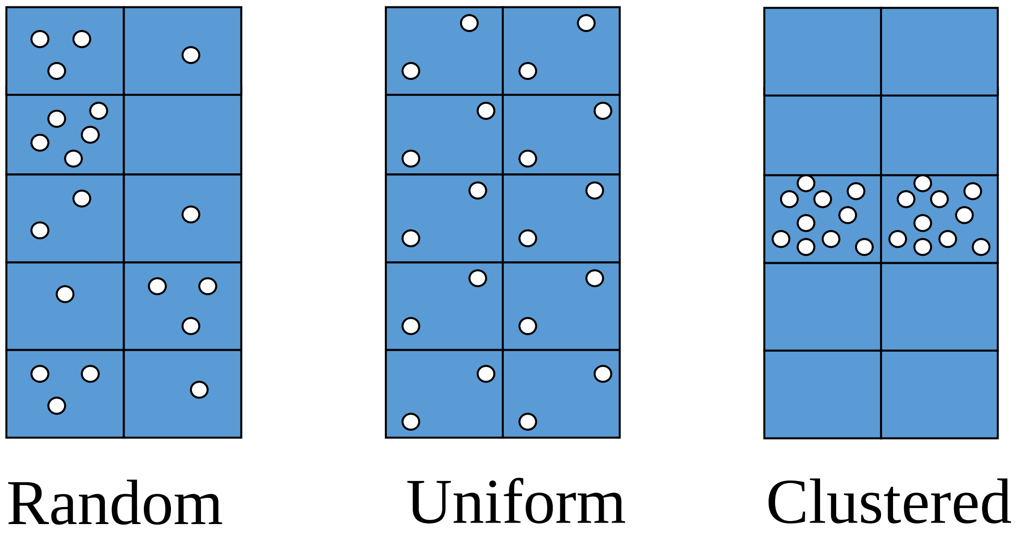
Inferential Spatial Statistics: Point Pattern Analysis

- **What is the purpose of Point Pattern Analysis?**
  - Test hypotheses about spatial patterns of points
- **What do we mean by spatial patterns?**
  - Mainly three types of spatial patterns
- **Why spatial patterns matter?**
  - Often, we cannot observe the spatial process, so we have to infer the process by observing the spatial patterns
  - Nonrandom ← Spatial processes
    - First order effect
    - Second order effect



Three types of spatial patterns

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First Order Effect

- **Diseases cluster (e.g., cancer, cold, coronavirus)**
- **First order effect describes Spatial Heterogeneity**
  - Variation in the study area (uneven distribution)
  - The probability of a spatial phenomenon occurring at each location varies throughout a study region
    - Diseases cluster
    - Robberies cluster

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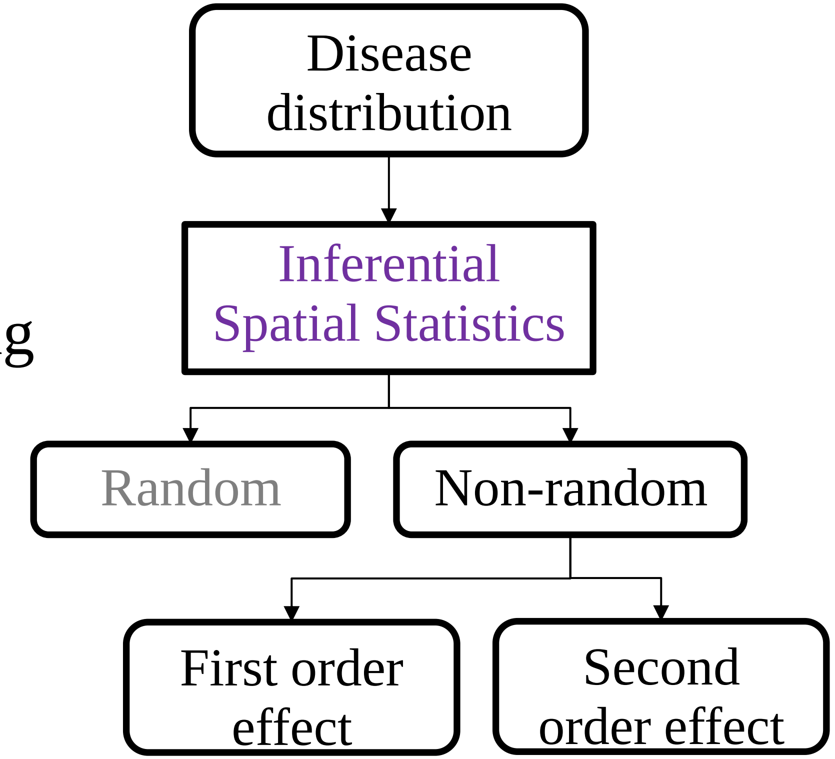
Second Order Effect

- **Diseases cluster (e.g., cancer, cold, coronavirus)**
- **Second order effect describes Spatial Interdependence**
- The occurrence of an event at one location increases the probability of the same event occurrence at the neighboring location, sometimes resulting in clusters across a study region.
  - Interdependence between points themselves
    - Diseases cluster because people catch them from others who have the disease (e.g., cold)



Exploring the cause of a disease

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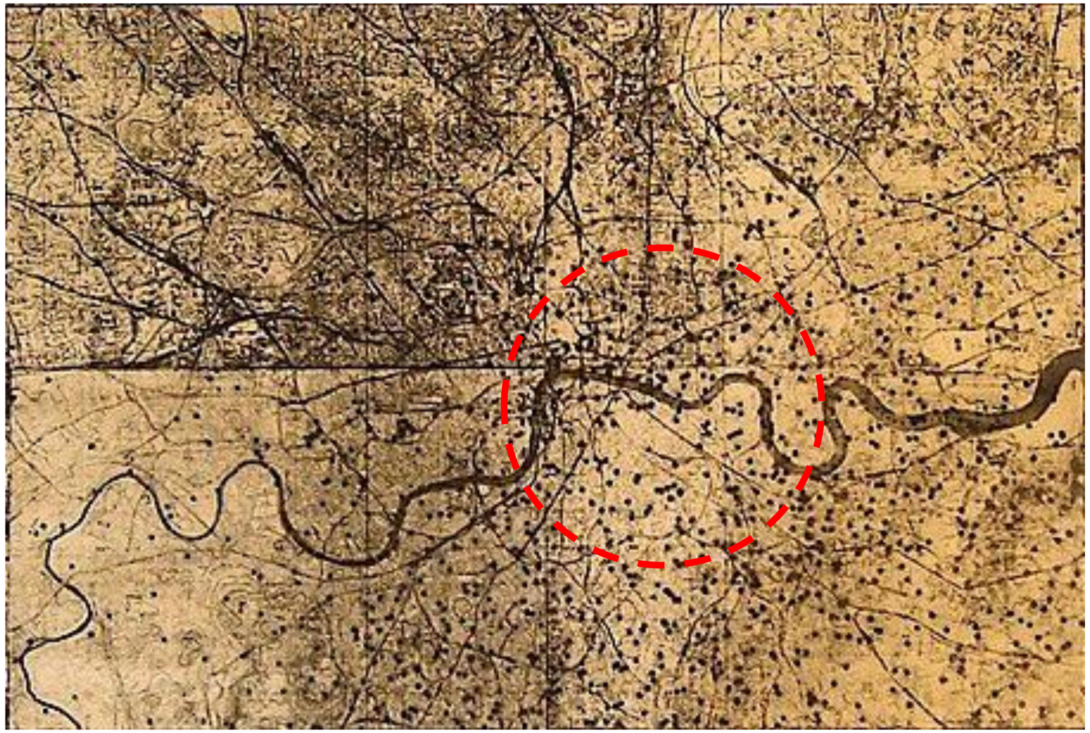
Point Pattern Analysis

- German launched 1,358 V-2 Rockets at London during World War II
- At first, its guidance systems were thought to be too primitive to hit specific targets
- However, according to the bomb damage maps, the impact sites seemed (visually) to be clustered
- If the rocket strikes were spatially clustered, the guidance systems must have been improved
- It was essential to assess if the impact sites were spatially clustered quantitatively



Distribution of V-2 Rocket Strikes on Central London, 1944

Exploratory Spatial Data Analysis (ESDA) method

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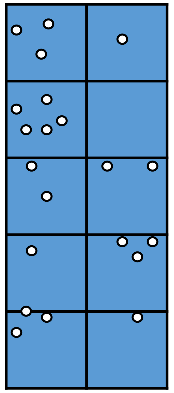
Point Pattern Analysis

- Approaches to point pattern analysis
  - Density-based approaches
  - Distance-based approaches
- **Density-based approaches**
  - Quadrat Analysis
  - Kernel Density Estimation



Construct grids

3	1
5	0
2	2
1	3
3	1

Variance
Mean
Var/Mean

Calculate variance and mean, and create the variance-mean ratio

Threshold values:

Uniform : Var/Mean ≈0
Random : Var/Mean ≈1
Clustered : Var/Mean >1

Determine spatial distribution

- **Quadrat Analysis**

The biggest problem is selecting the suitable quadrat size **(Modifiable Area Unit Problem)**

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Point Pattern Analysis

- Approaches to point pattern analysis
  - <u>Density</u>-based approaches
  - <u>Distance</u>-based approaches
- **Density-based approaches**
  - Quadrat Analysis
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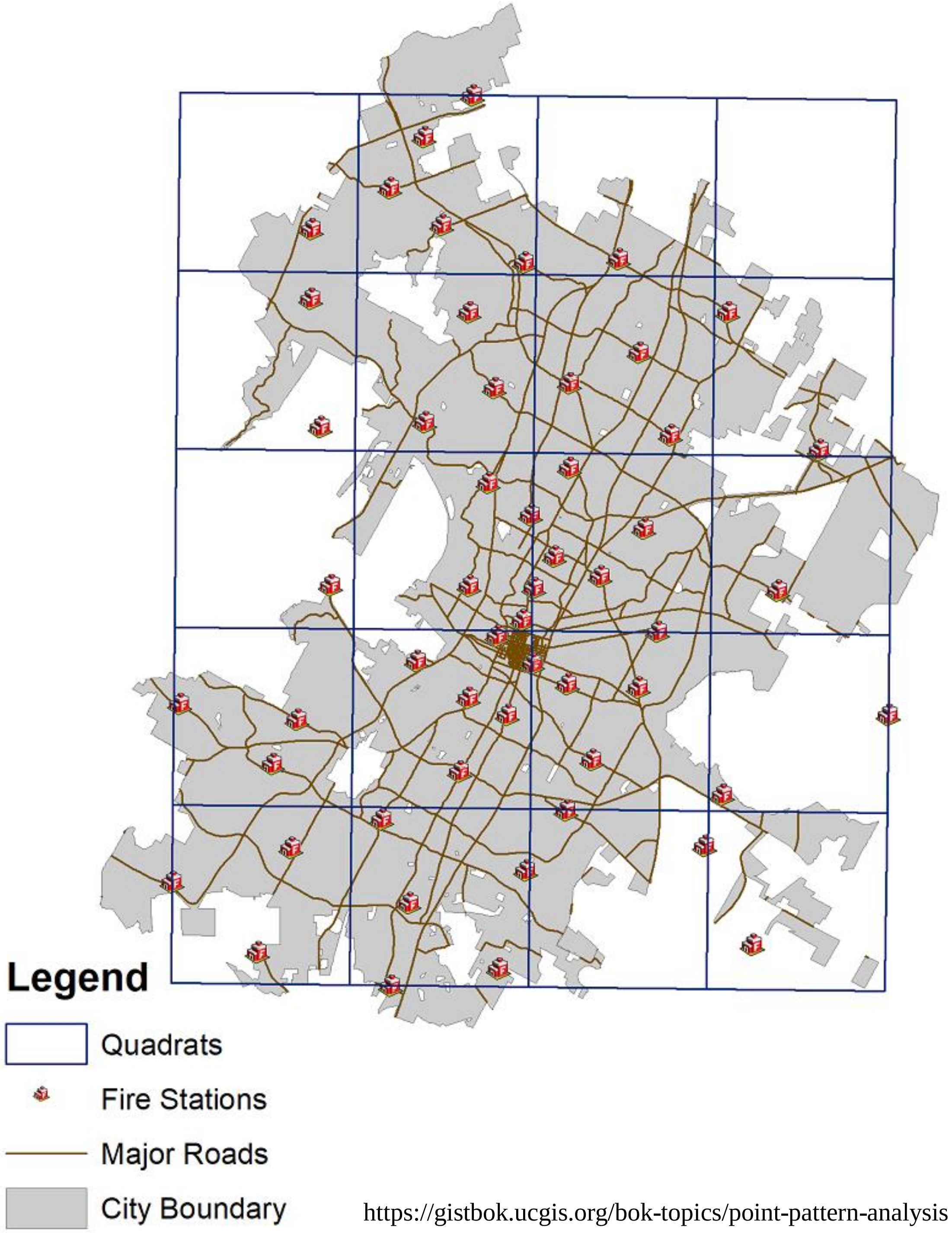

https://gistbok.ucgis.org/bok-topics/point-pattern-analysis

Point Pattern Analysis

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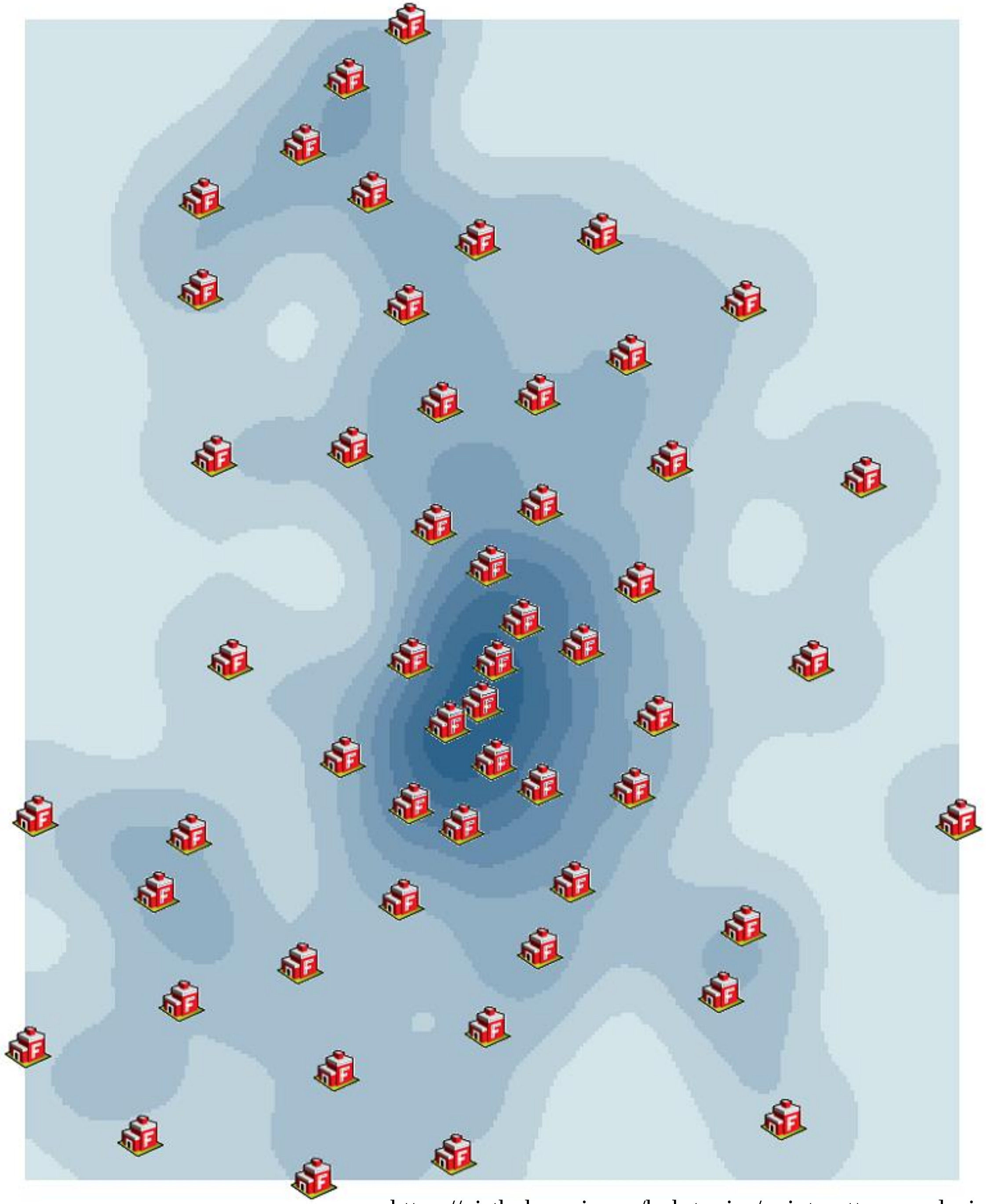

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Point Pattern Analysis

- Approaches to point pattern analysis
    - Density-based approaches
    - Distance-based approaches
- **Distance-based approaches**
    - **Nearest Neighbor Index (NNI)**
    - G function
    - F function
    - Ripley's K function

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Point Pattern Analysis

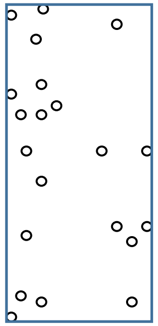
- Distance-based approaches
  - **Nearest Neighbor Index (NNI)**

- **Calculate NNI**

1. The mean of the distance <u>observed</u> between each point and its nearest neighbor
2. The <u>expected</u> mean distance <u>if</u> the distribution is random

$$NNI = \frac{Observed\ mean\ distance}{Expected\ mean\ distance}$$

$$Expected\ mean\ distance = 0.5\sqrt{\frac{Area}{Number\ of\ points}}$$



Point	Nearest Point	Distance
1	2	1
2	3	0.1
3	2	0.1
…	…	
20	21	1
21	19	0.4

Calculating the <u>nearest neighbor</u> distance

Threshold values:

- Random pattern, NNI ≈ 1
- Clustered pattern, NNI ≈ 0
- Uniform pattern, NNI ≈ 2.149

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– Spatial Autocorrelation

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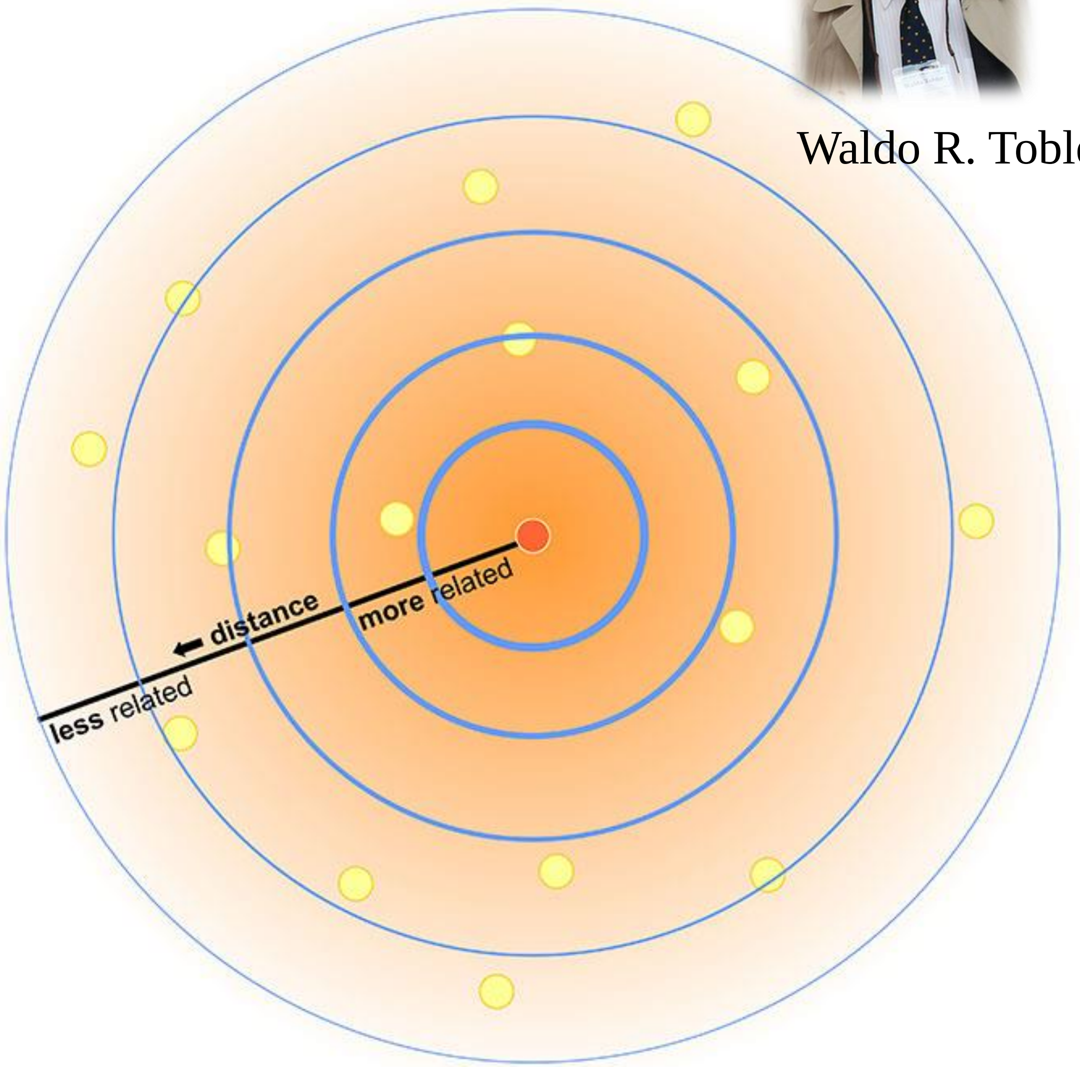
Spatial Autocorrelation

Waldo R. Tobler

- Spatial Autocorrelation (SA) is the most important concept in Spatial Statistics
- Definition of Spatial Autocorrelation
  - The <u>degree</u> to which characteristics at one location are similar (or dissimilar) to those nearby
- **It is the confirmation of Tobler's first law of geography**
  - *Everything is related to everything else, but near things are more related than distant things*



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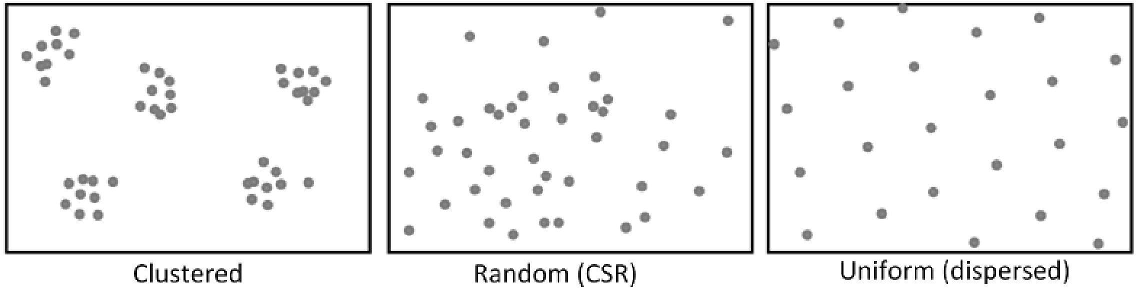
Spatial Autocorrelation

- **Differences and similarities between Point Pattern Analysis and SA**

➢ *Point Pattern Analysis*

✓ Points only

✓ Only location, no "magnitude" value

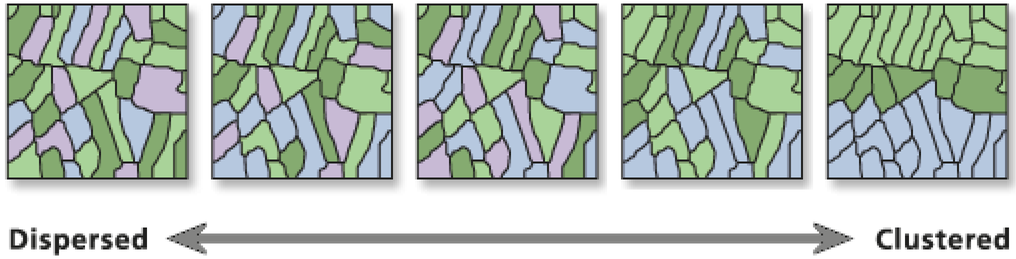


➢ *Spatial Autocorrelation*

✓ Points and polygons

✓ Location and attribute variable

❖ Income, GDP, rainfall, crime rate etc.



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Spatial Autocorrelation

The name explains itself

Spatial:

Location-related,

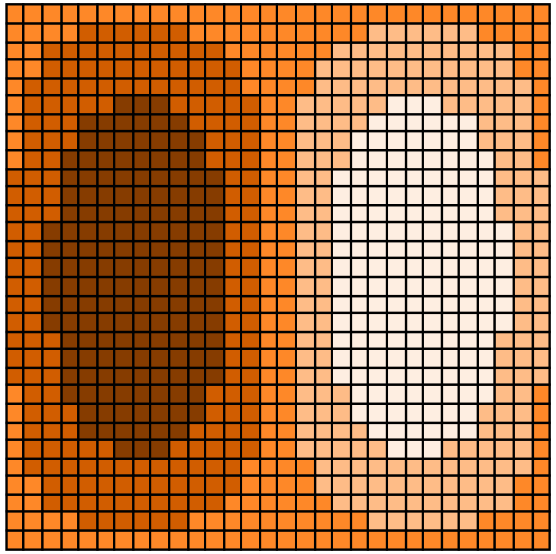
On a map

Auto:

Self (Interdependence)

Correlation:

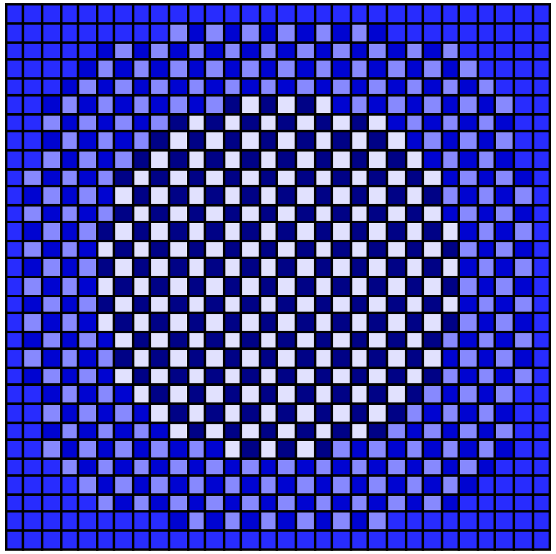
Degree of relative similarity



Positive Spatial Autocorrelation
Similar values cluster together on a map

White: High value

Dark: Low value



Negative Spatial Autocorrelation
Dissimilar values cluster together on a map

Source: Dr Dan Griffith

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Why Is Spatial Autocorrelation Important?

- **Two reasons**

1. **It <u>implies the existence of spatial process</u>**
   - Why are near-by things similar to each other?
   - Why do high-income people live “next door” to each other?
   - Why are rich cities surrounded by other rich cities?
   - All these **geographical** questions are related to **locations**, and can be explained by **Spatial Autocorrelation**

Other examples?

1. **Nearby things are similar to each other**
2. **Nearby things are dissimilar to each other**

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Why Is Spatial Autocorrelation Important?

- **Two reasons**

1. **Spatial Autocorrelation <u>invalidates many traditional statistical inference assumptions</u>**
    - Traditional statistical inference often assumes that values of observations in each sample are <u>**independent**</u> of one another
    - However, if SA exists, the assumption is wrong
    - In this case, <u>spatial statistics</u> should be used

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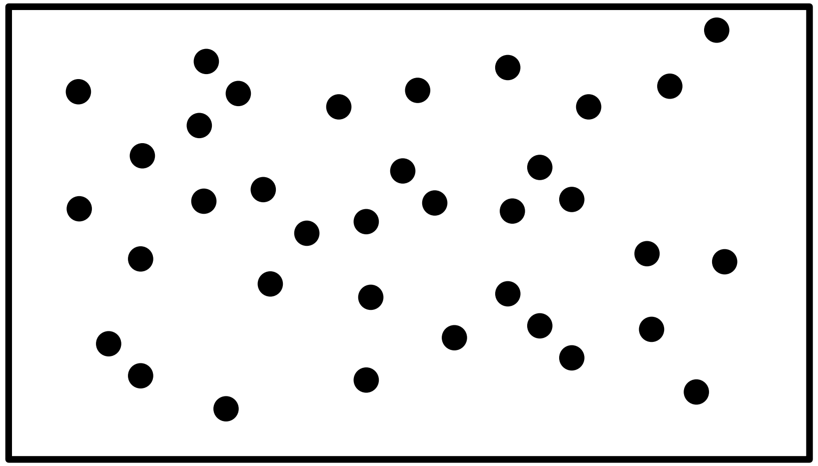
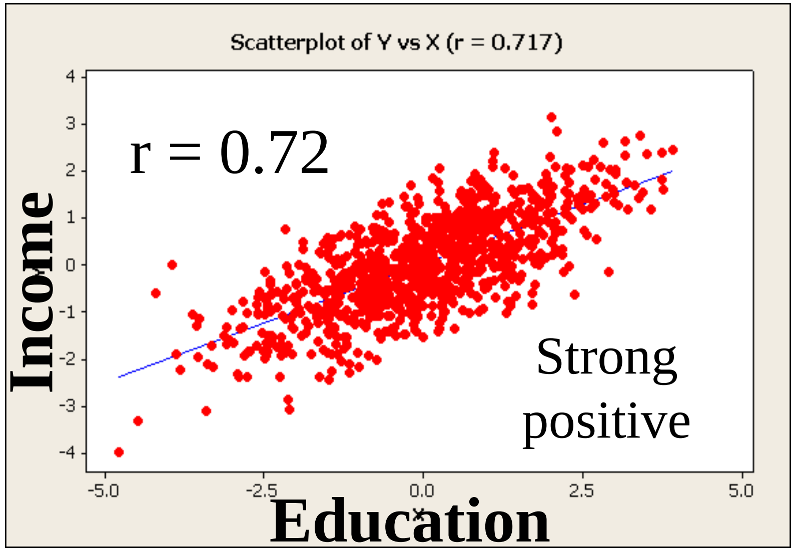
How Does SA Impact Standard Statistics?

- Correlation analysis between Income and Education by **Standard Statistics**



Sample collection locations



Correlation Analysis

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Measuring Spatial Autocorrelation

- Spatial Autocorrelation is measured based on the *Spatial Weight Matrix (SWM)*
- The *SWM* measures the relative location between each pair of points (or polygons)
- The *SWM* is a representation of the spatial structure of your data
- Two methods for measuring *Spatial Weight Matrix*
  - Weights based on contiguity – **binary** (0, 1)
  - Weights based on distance – **continuous** variable

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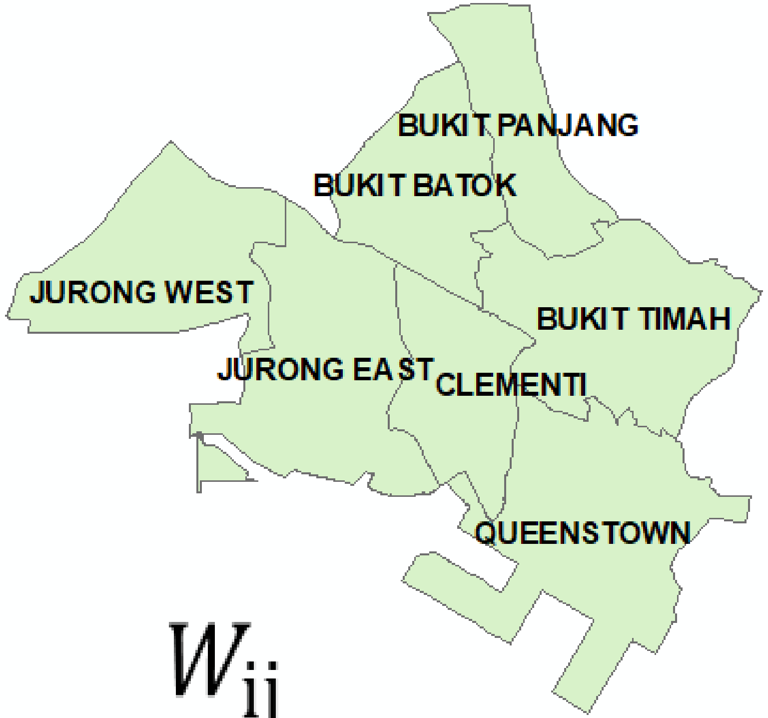
Contiguity-based SWM

	BUKIT BATOK	BUKIT PANJANG	BUKIT TIMAH	CLEMENTI	JURONG EAST	JURONG WEST	QUEENSTOWN
BUKIT BATOK	*						
BUKIT PANJANG	1	*					
BUKIT TIMAH	1	1	*				
CLEMENTI	1	0	1	*			
JURONG EAST	1	0	0	1	*		
JURONG WEST	0	0	0	0	1	*	
QUEENSTOWN		0	1	1	0	0	*



W_{ij}

Contiguity-based Spatial Weight Matrix

If in adjacency relation 1, else 0

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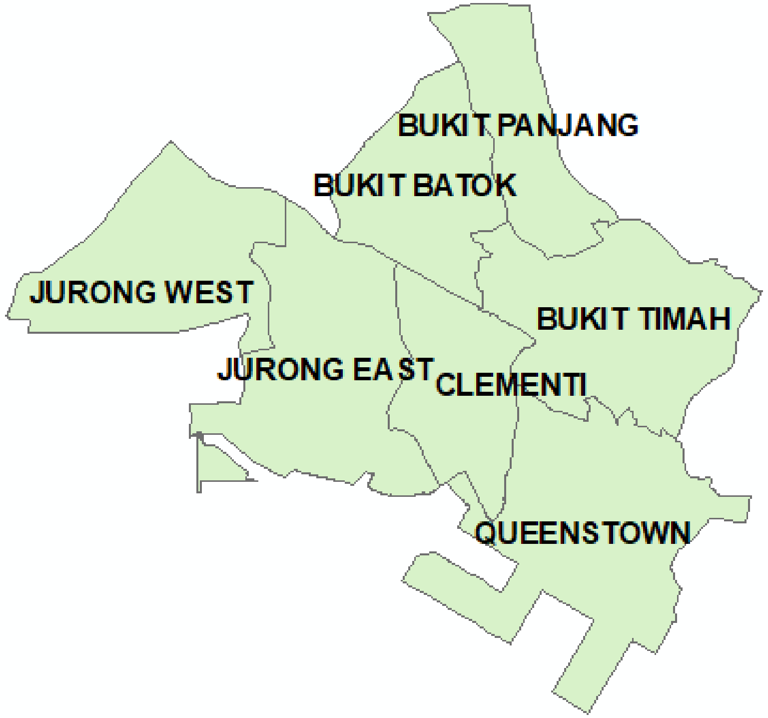
Distance-based SWM

	BUKIT BATOK	BUKIT PANJANG	BUKIT TIMAH	CLEMENTI	JURONG EAST	JURONG WEST	QUEENSTOWN
BUKIT BATOK	*						
BUKIT PANJANG	0.42	*					
BUKIT TIMAH	0.20	0.22	*				
CLEMENTI	0.23	0.18	0.28	*			
JURONG EAST	0.22	0.15	0.16	0.34	*		
JURONG WEST	0.17	0.12	0.10	0.15	0.25	*	
QUEENSTOWN	0.12	0.12	0.22	0.24	0.15	0	*



Distance-based Spatial Weight Matrix

The weight between two polygons is inversely proportional to the distance between them

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Global Measures of Spatial Autocorrelation

- Moran's I is the most common measure of Spatial Autocorrelation

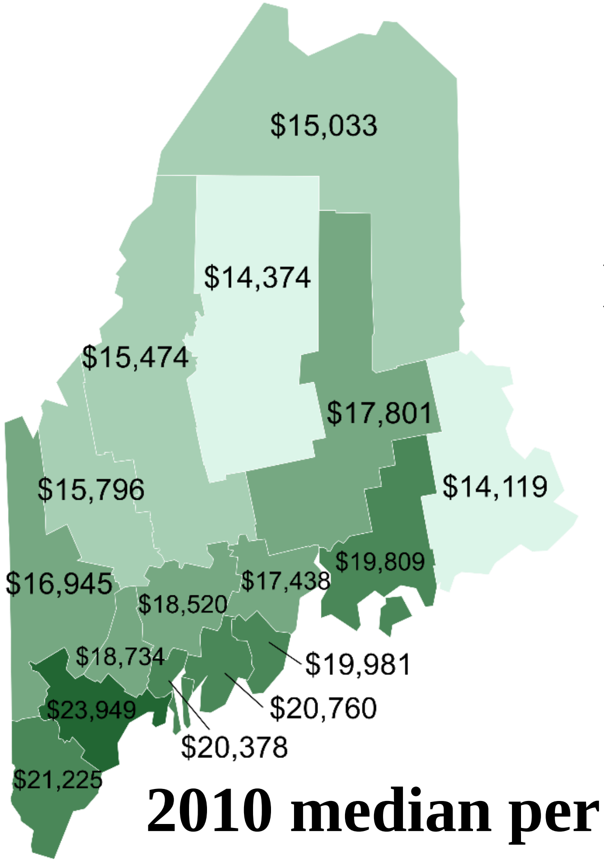
+1 High positive spatial autocorrelation

0 No spatial autocorrelation

-1 High negative spatial autocorrelation

Patrick Alfred Pierce Moran

Australian



Moran's I = 0.377

Positive SA

2010 median per capita income for Maine, USA

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Global Measures of Spatial Autocorrelation

- Geary's C is similar to Moran's I

2 — High negative spatial autocorrelation

1 — No spatial autocorrelation

0 — High positive spatial autocorrelation

Roy C. Geary

- **Comparison of Moran's I and Geary's C**
  - ➢ In general, they result in similar conclusions
  - ➢ Moran's I is often preferred due to the greater general stability

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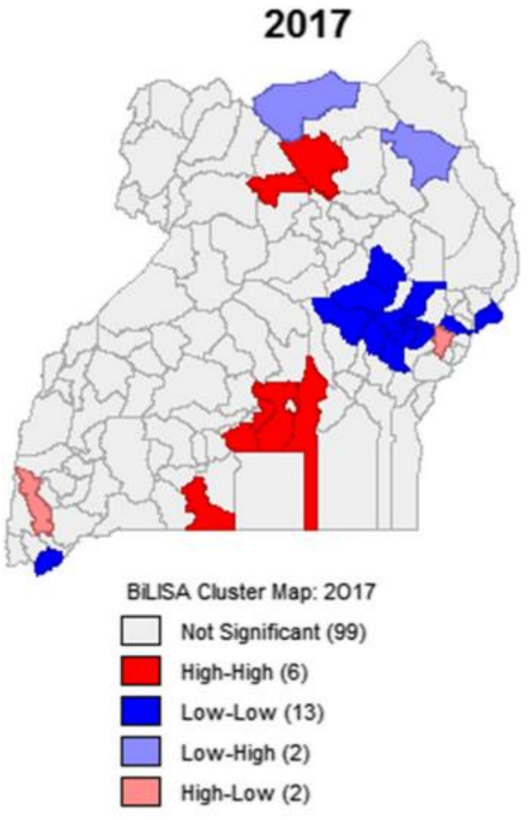
Hot Spots and Cold Spots

- Four types of <u>Spatial Clustering Patterns</u>
  - Positive Spatial Autocorrelation
    - High-High clustering (<u>High</u> values cluster together, **Hot Spots**)
    - Low-Low clustering (<u>Low</u> values cluster together, **Cold Spots**)
  - Negative Spatial Autocorrelation
    - Low-High clustering (<u>Low</u> values surrounded by <u>high</u> values)
    - High-Low clustering (<u>High</u> values surrounded by <u>low</u> values)



Clustering of TB and HIV across Uganda

- **Both Moran's I and Geary's C can <u>only distinguish</u> positive or negative spatial autocorrelation**
- **<u>Cannot tell</u> if these are hot spots or cold spots**

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Getis-Ord General G-Statistic

- The general G statistic can distinguish between hot spots and cold spots
  - G is relatively <u>large</u> → potential <u>hot spots</u>
  - G is relatively <u>low</u> → potential <u>cold spots</u>

Arthur Getis

- General G will **NOT** show <u>negative</u> spatial autocorrelation
- General G can <u>only</u> use <u>distance-based</u> Spatial Weight Matrix
- General G should <u>only</u> be calculated for <u>ratio scale</u> data
  - It allows you to compare two values using ratio
    - E.g., crime rates & birth rates

J. Keith Ord

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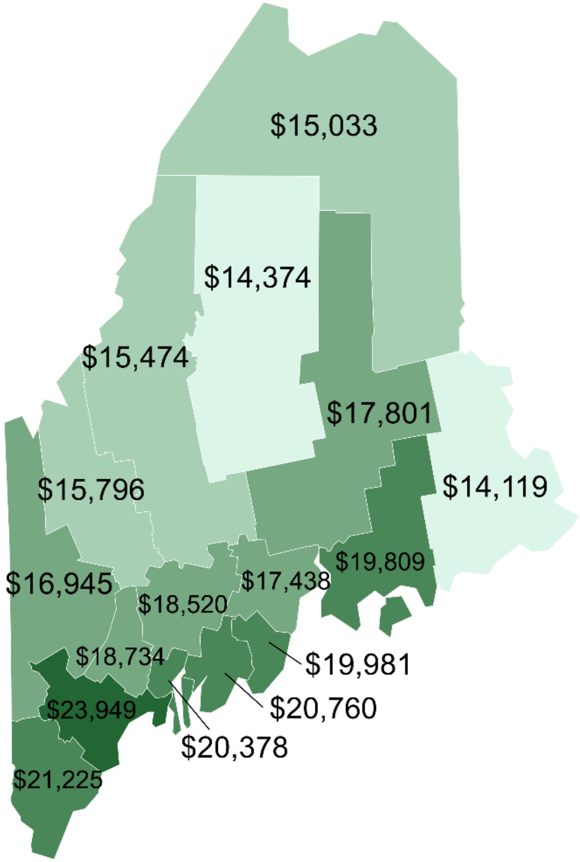
Local Indicators of Spatial Association (LISA)

- Global measures of Spatial Autocorrelation are a measure for the entire area
- Local measures are needed when we want to look at the SA at each location or observation unit
- Why local measures matter?
  - Global measures indicate **No** SA, but **local** SA exists
  - **Positive** and **negative** SA exist **simultaneously** on the same map
  - The above issues are caused by **spatial heterogeneity**



What are the SA effects at each county of Maine?

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Local Indicators of Spatial Association (LISA)

- LISA was proposed by Luc Anselin in 1995, therefore, LISA is often called Anselin's LISA
  - Luc Anselin 1995. *Local Indicators of Spatial Association-LISA*. Geographical Analysis 27: 93 -115

Luc Anselin

Local indicators of **spatial association**—LISA
L Anselin - Geographical analysis, 1995 - Wiley Online Library
… where A is a global **indicator** of **spatial association** and 7 is a scale factor. In other words, the sum of the **local indicators** is proportional to a global **indicator**. For the latter, a statement …
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THE UNIVERSITY OF CHICAGO | DEPARTMENT OF SOCIOLOGY

Luc Anselin

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Luc Anselin is a native of Belgium, where he did undergraduate work and a master's degree in economics and econometrics at the Free University of Brussels (VUB). His PhD is from Cornell University in the interdisciplinary field of Regional Science.

Anselin comes to the University of Chicago from Arizona State University where he was a Regents' Professor and held the Walter Isard Chair. He was the founding Director of the School of Geographical Sciences and Urban Planning. He also started and directed the GeoDa Center for Geospatial Analysis and Computation. Before ASU, he was a Professor at the University of Illinois, Urbana-Champaign, where he directed the

https://sociology.uchicago.edu/directory/Luc-Anselin

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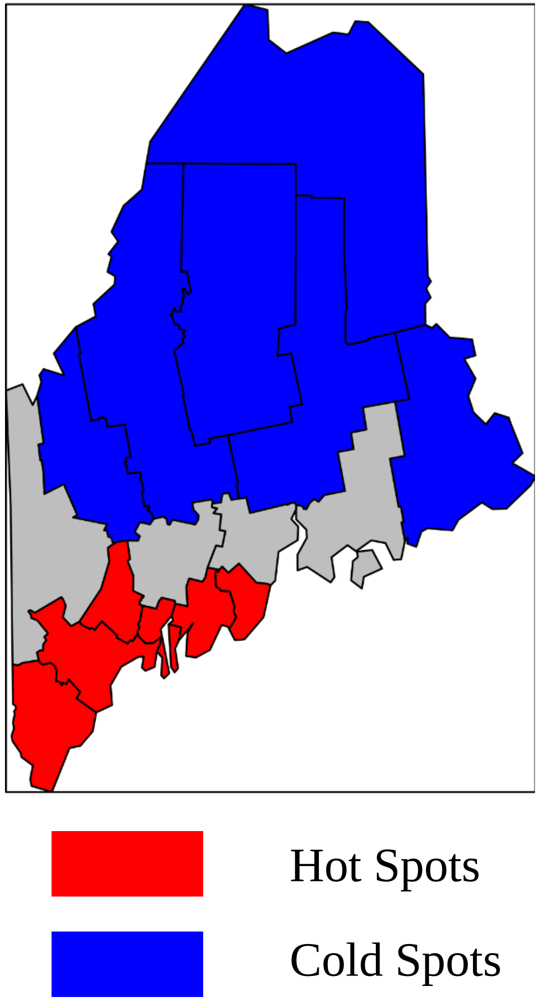
Identifying Hot and Cold Spots with LISA

- Global measures can only tell us there are Hot or Cold Spots in the whole area
- But LISA can tell us where the Hot and Cold Spots are



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Local Indicators of Spatial Association (LISA)

- Moran's I, Geary's C, and Getis-Ord General G are the global measures, and they all have their local versions
  - Local Moran's I (https://pro.arcgis.com/en/pro-app/latest/tool-reference/spatial-statistics/cluster-and-outlier-analysis-anselin-local-moran-s.htm)
  - Local Geary's C
  - Local Getis-Ord G (https://pro.arcgis.com/en/pro-app/latest/tool-reference/spatial-statistics/hot-spot-analysis.htm)

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Demo: Identifying Hot Spots

- **Exploring 911Call data using Hot Spot Analysis**

Where are the Hot Spots of 911Calls

Distributions and numbers of 911Calls in Portland, Oregon

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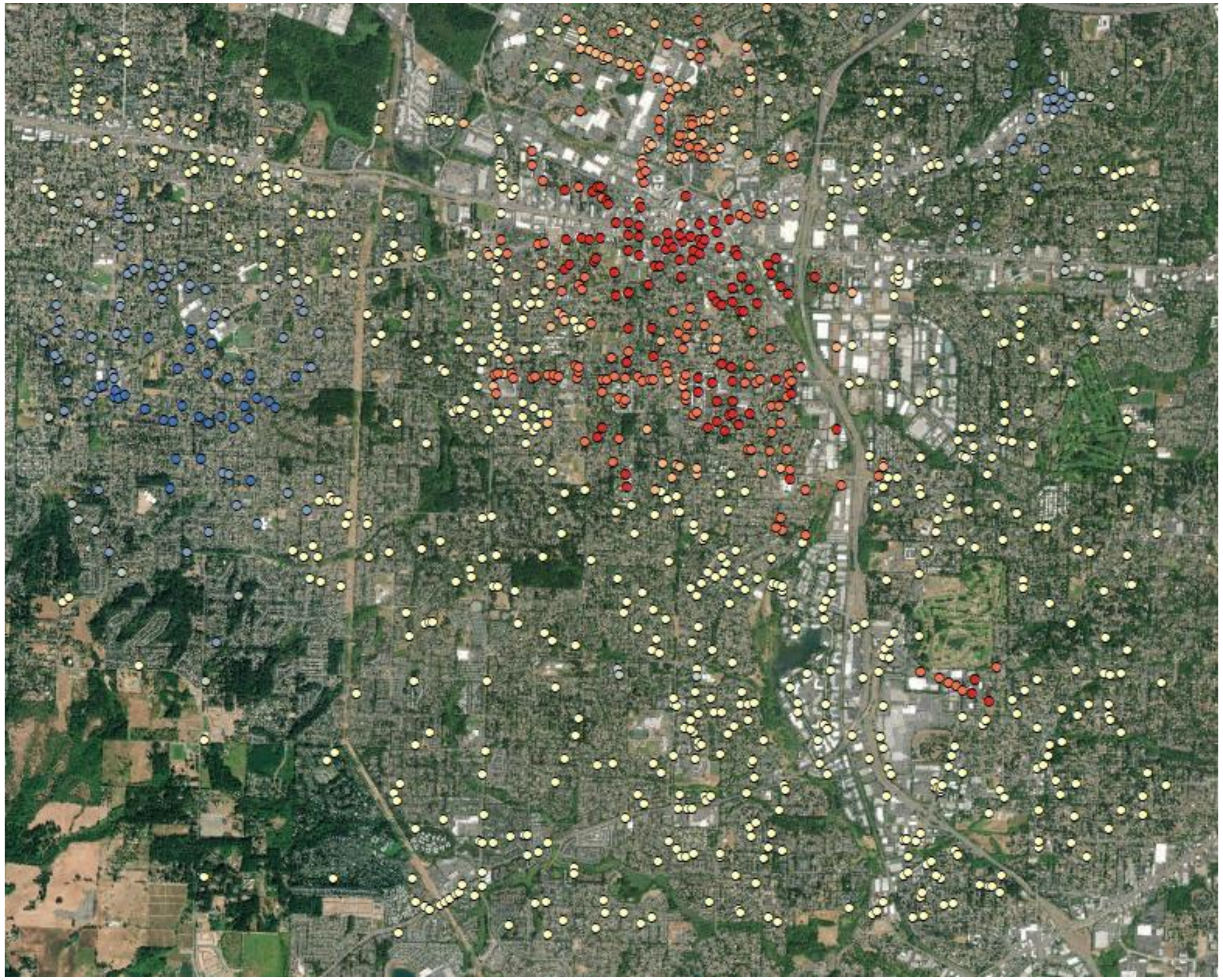
Demo: Identifying Hot Spots

Gi_Bin
- Cold Spot - 99% Confidence
- Cold Spot - 95% Confidence
- Cold Spot - 90% Confidence
- Not Significant
- Hot Spot - 90% Confidence
- Hot Spot - 95% Confidence
- Hot Spot - 99% Confidence



Red: Hot Spots

Blue: Cold Spots

Use IDW to estimate values between points

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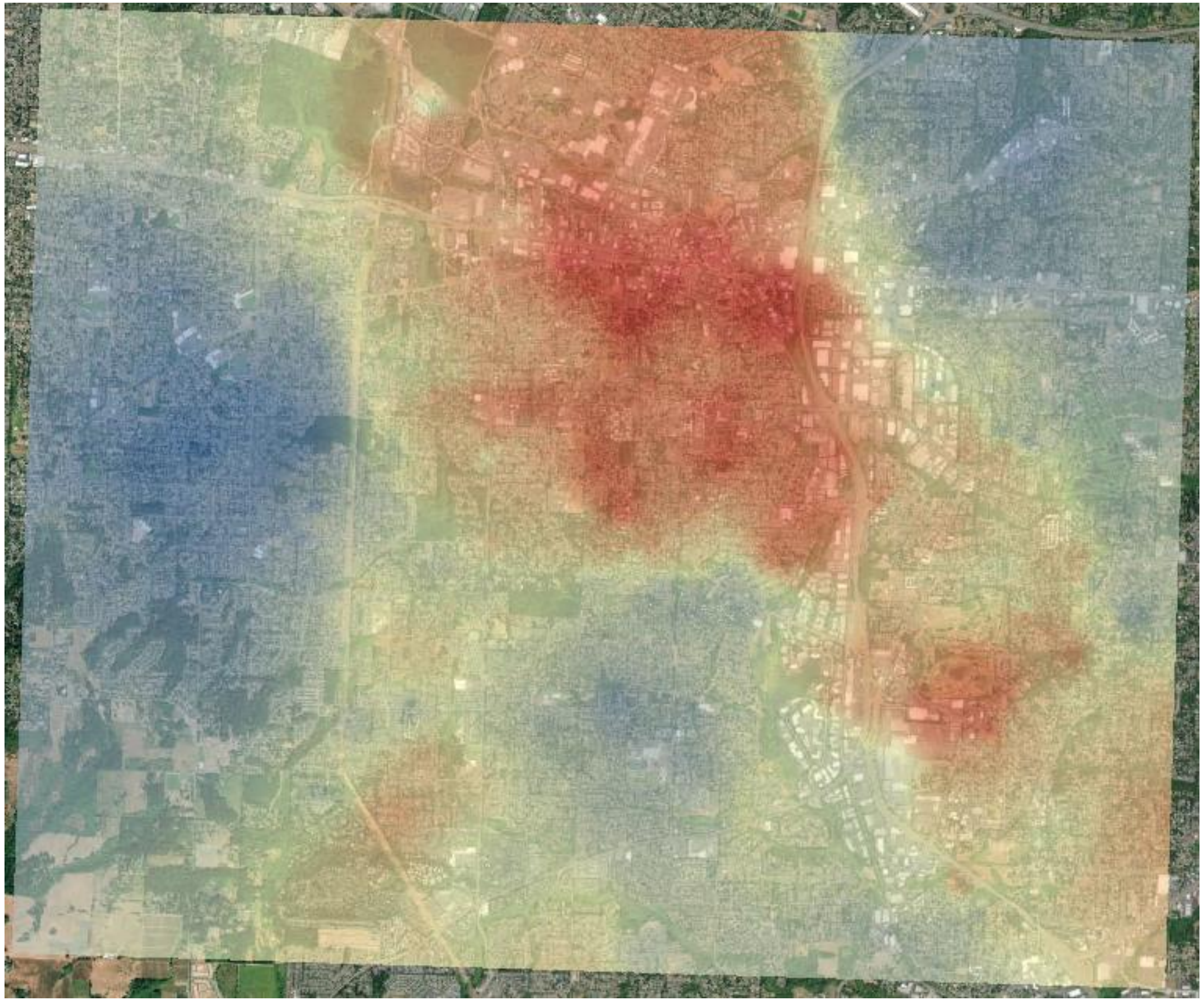
Demo: Identifying Hot Spots

IDW Interpolation



Red indicates potential hot spots

Blue indicates potential cold spots

- **What does the map imply?**
- **How can we apply the Hot Spot map?**

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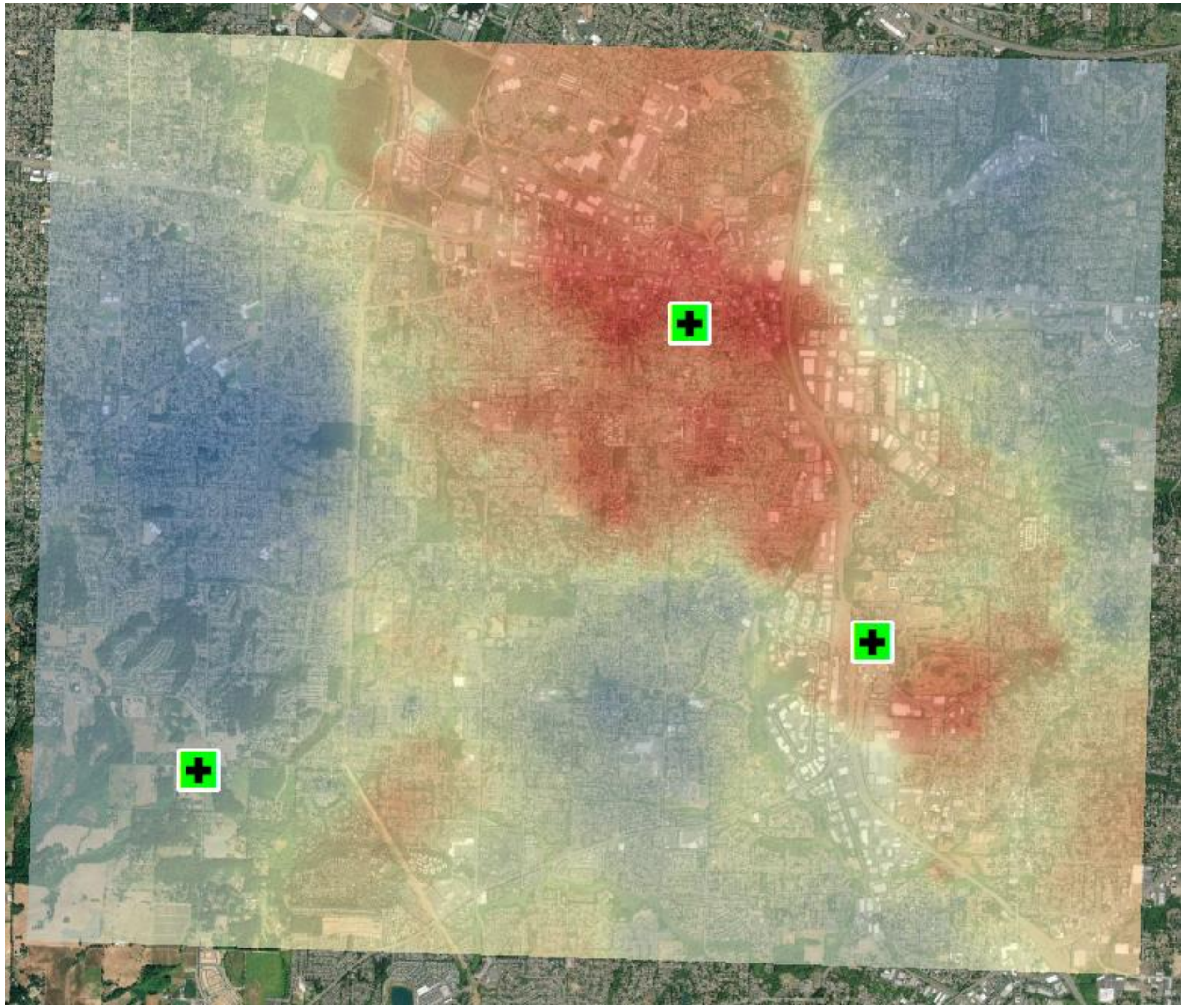
Demo: Identifying Hot Spots

Where should the Response Stations be located?



Response Stations

Please watch the full demo here: https://storymaps.arcgis.com/stories/42313404667341c3a6b33ce5f73573e6

The relevant data and lab handout are available upon request

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Software for Spatial Statistics

- ArcGIS (licensed)
- QGIS
- GeoDA (free)
  - Designed by Luc Anselin, Arizona University
- R software (free)
- Python
  - PySAL: Python Spatial Analysis Library

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Summary

- **Descriptive and Inferential Standard Statistics**
- **What is spatial statistics**
  - Descriptive and Inferential Spatial Statistics
    - Spatial data: point, polyline, polygon and continuous data
- **Point pattern analysis**
  - Spatial patterns: Random, Uniform and Clustered distributions
  - Spatial processes: First and second order effects (Spatial heterogeneity and interdepedence)

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Summary

- **Spatial autocorrelation**
  - Global measures: Global Moran's I, Geary's C, and Getis-Ord general G
    - Spatial Weight Matrix: contiguity-based and distance-based
  - Local Indicators of Spatial Association (LISA)
    - Identifying Hot Spots and Cold Spots

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THANK YOU

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