NUS National University of Singapore

GE2215 Lecture 6 Geometric Transformation

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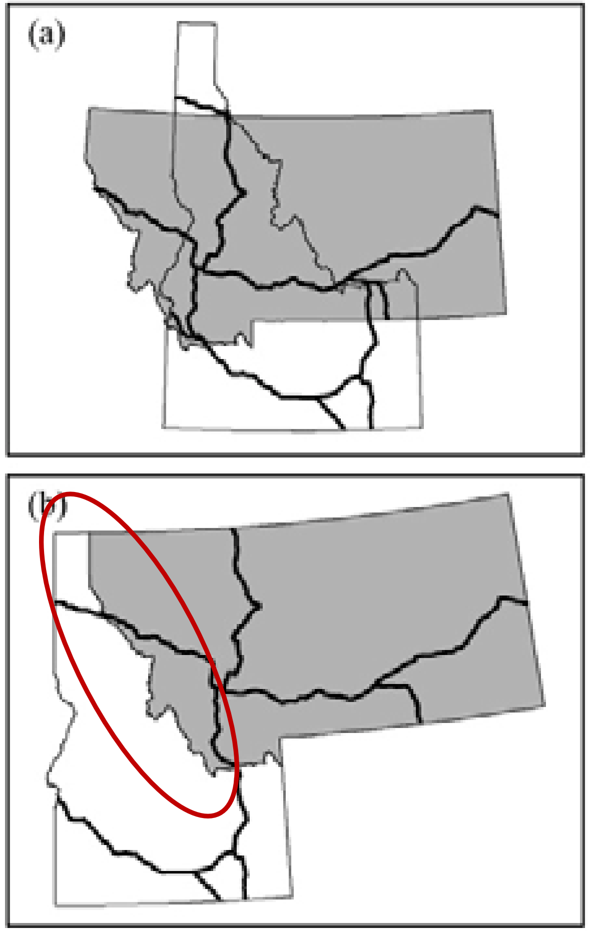
Recap: Why coordinate systems matter?

- It is used for:
  - **Talking about** locations and spatial measurements
  - **Creating** a new set of spatial data (e.g., GPS)
  - **Acquiring** spatial data from other data sources (e.g., an existing geodatabase)
  - **Overlaying/Displaying** two or more map layers. They are not going to register spatially unless they are based on the same coordinate system

Recap: Why coordinate systems matter?



- Interstate highways in Idaho and Montana based on different coordinate systems

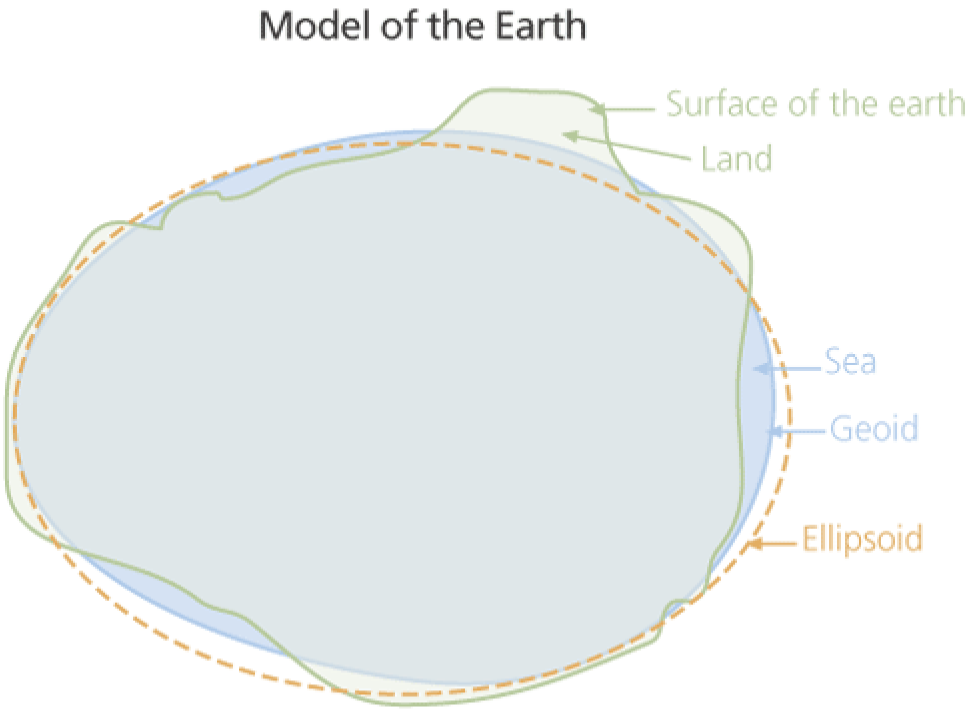
- Interstate highways in Idaho and Montana based on the same coordinate systems

Recap: Geographic Coordinate Systems

- The geographic coordinate system (**GCS**) is defined by **longitude** and **latitude**
- Latitude range
- Longitude range
- Parallels and Meridians
- What shape is the earth?
  - Earth surface
  - Geoid
  - Earth ellipsoid

Simplification

Approximation

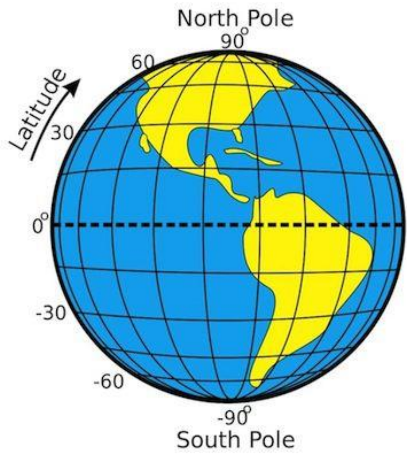


Recap: Datum

- Datum
  - A datum = an ellipsoid + an origin
  - Local datum and geocentric datum
- Why do we need a local datum?
- WGS 84 datum – Used by all GPS satellites
- Singapore datum – SVY21 datum

Recap: Why map projection?

- Geographic coordinates are spherical coordinates represented by longitudes and latitudes. It is not easy to calculate the **distance**, **direction** and **area** on a curve surface.
- The commonly used maps are **plane-based**, which accord with people's visual and psychological perceptions, and are convenient for the above measurements.
- The curved surface of the earth is **not spreadable**
- Map projection is needed to turn a curve surface to a plane surface



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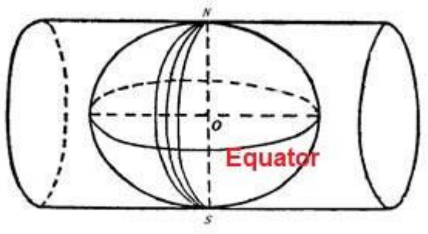
Recap: Universal Transverse Mercator (UTM) PCS

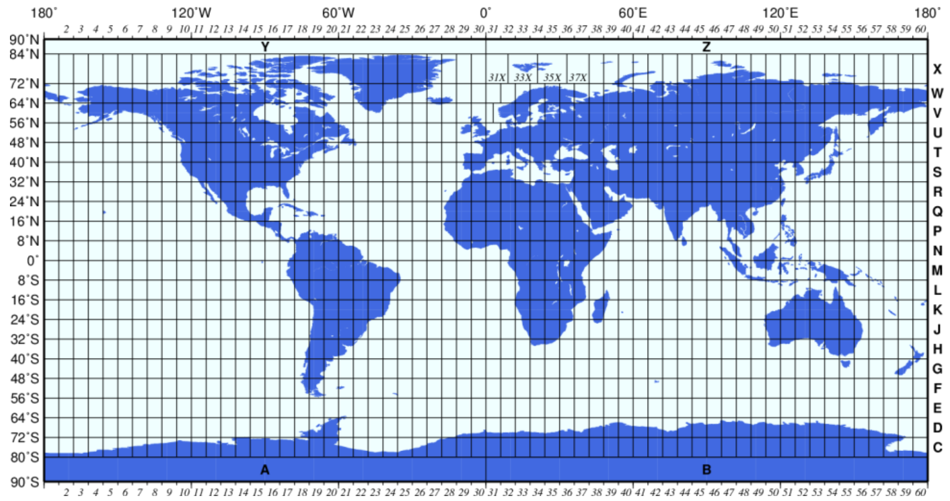


- Transverse
- Cylindrical
- Equal-angle

Map scale

1/25,000 – 1/50,000 → **Six-degree** division is adopted. The whole earth is divided into **60** zones

>1/10,000 → **Three-degree** division is used. The whole earth is divided into **120** zones



Recap: Spatial coordinate transformation

- Often, the data sets we have are in different coordinate systems
- It is a good idea to transform them to the same coordinate system

Geographic coordinate → (**Projection**) → **Projected coordinate**

Projected coordinate system 1 → (**Reprojection**) → **Projected coordinate system 2**

The algorithms mathematical methods of projections and reprojections are not required to master. This can easily be done in most of GIS software

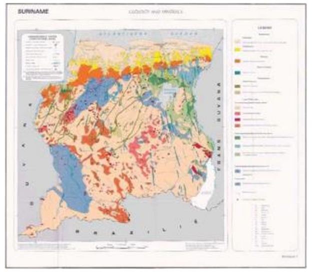
Outline of this lecture

- Background about geometric transformation
- Transformation method
- Control points
- Root Mean Square error
- Resampling

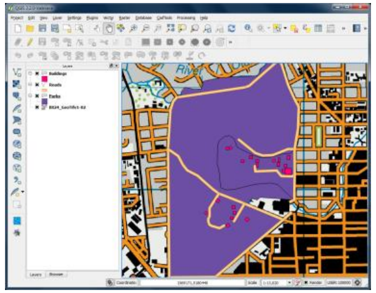
Why geometric transformation?

- Let's review the **map digitization** process first

Map scanning



Manual vectorization

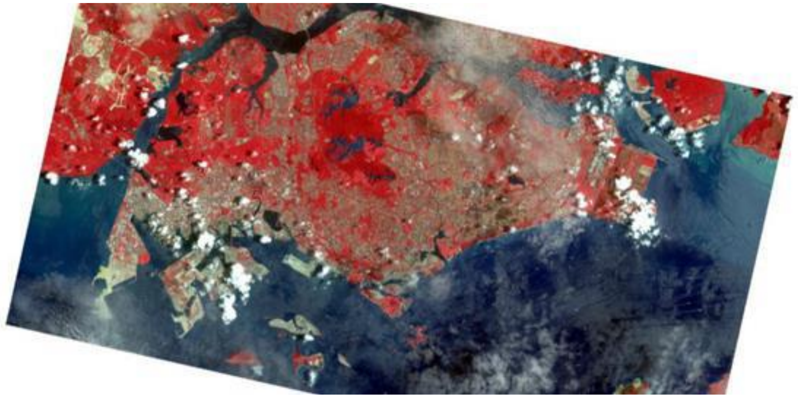


A paper map **A scanned map** **A digitized map**

- Both the scanned map (raster data) and the digitized map (vector data) cannot be aligned spatially with layers in GIS because they don't have spatial references

Why geometric transformation?

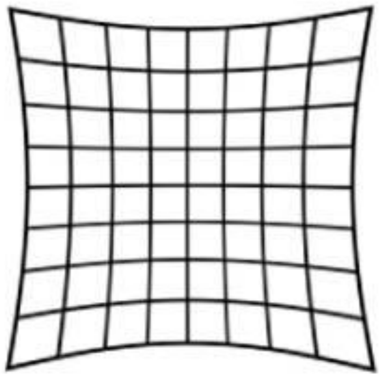
- Geometric transformation can be used to:
  - Assign **coordinate systems** to digitized **maps** and **images**
    - **Scanned paper maps**
    - **Remote sensing images**

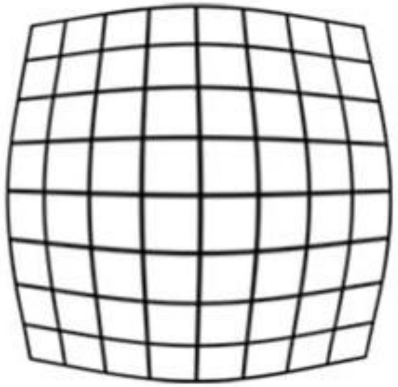


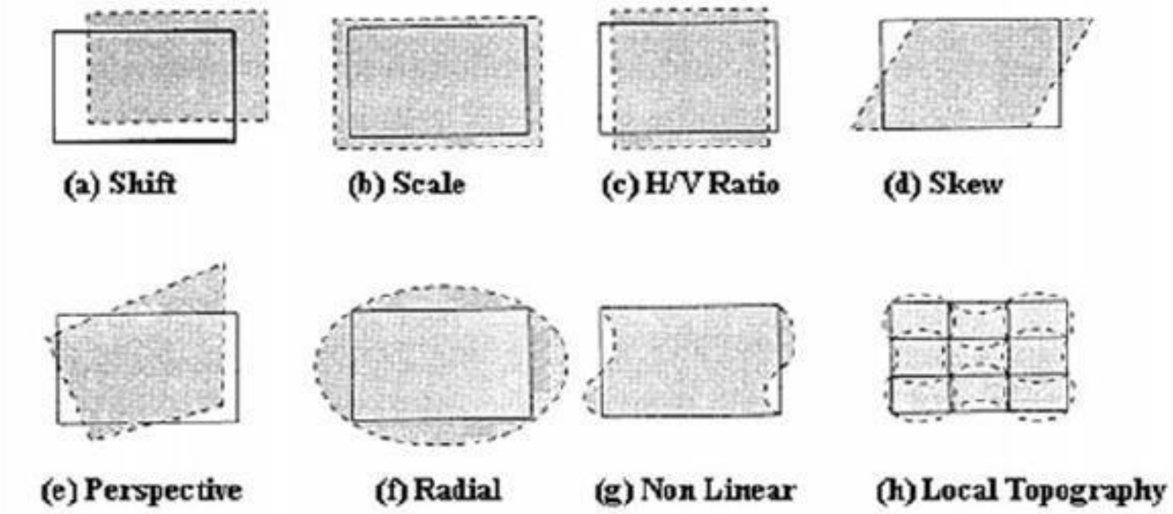
http://libmaps.nus.edu.sg/

Why geometric transformation?

- **Map** or **image** deformations are inevitable during the production process.
- Some maps and remote sensing images have high geometric precision, while some have low precision



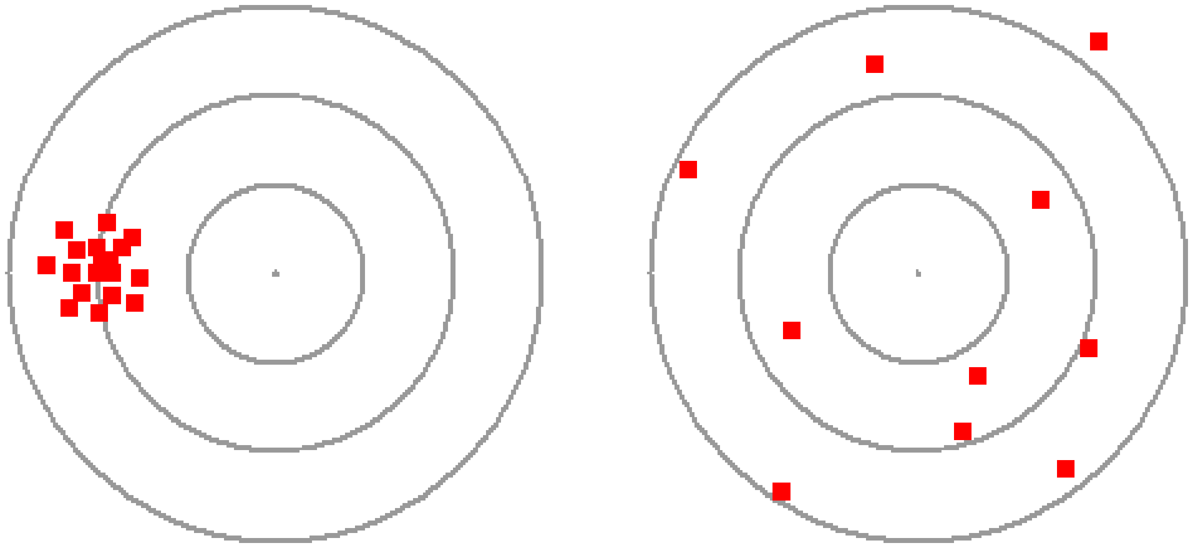




Assume we have one map (**Map #1**) with high geometric precision and another map (**Map #2**) with low precision. The precision of **Map #2** can be improved by **Map #1** and **geometric transformation**

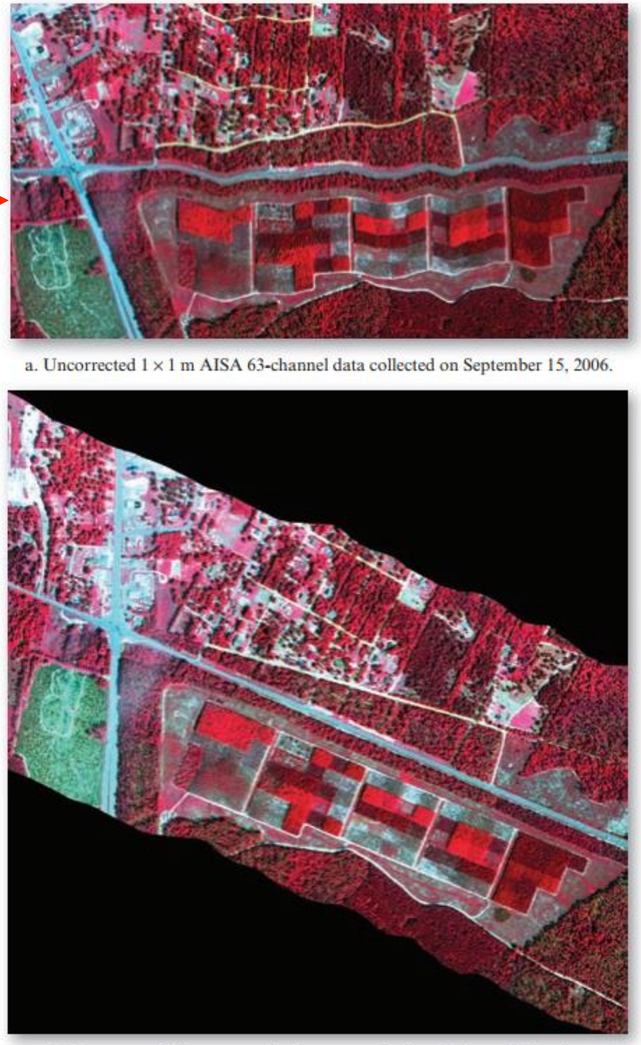
Why geometric transformation?

- Correct **random** **(non-systematic)** distortion (deformation)



Geometric Correction of Airborne Imaging Spectrometer for Applications (AISA) Imagery of Forest Experimental Plots near Aiken, SC

Map is distorted (random error)



a. Uncorrected 1 × 1 m AISA 63-channel data collected on September 15, 2006.

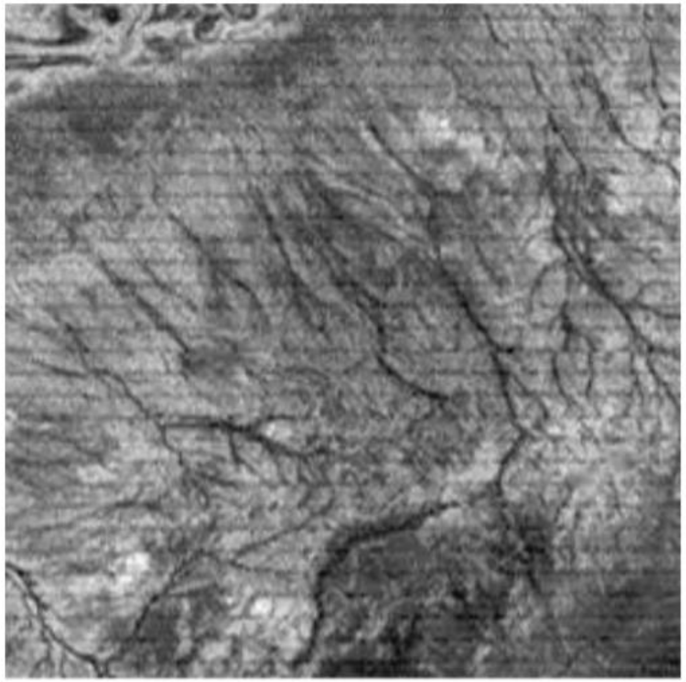
b. Hyperspectral data geometrically corrected using GCPs, a third-order polynomial and nearest-neighbor resampling.

Jensen (2016)

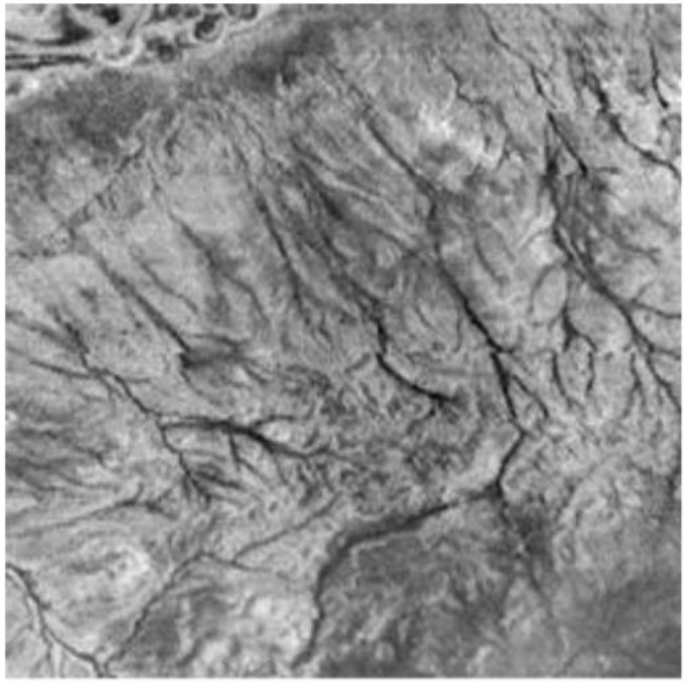
Systematic vs. Random distortions

- Systematic distortions
  - Predictable; can be corrected by mathematical formulas (**parametric**)
  - Often corrected during preprocessing

Original Band 1



Band 1 after destriping

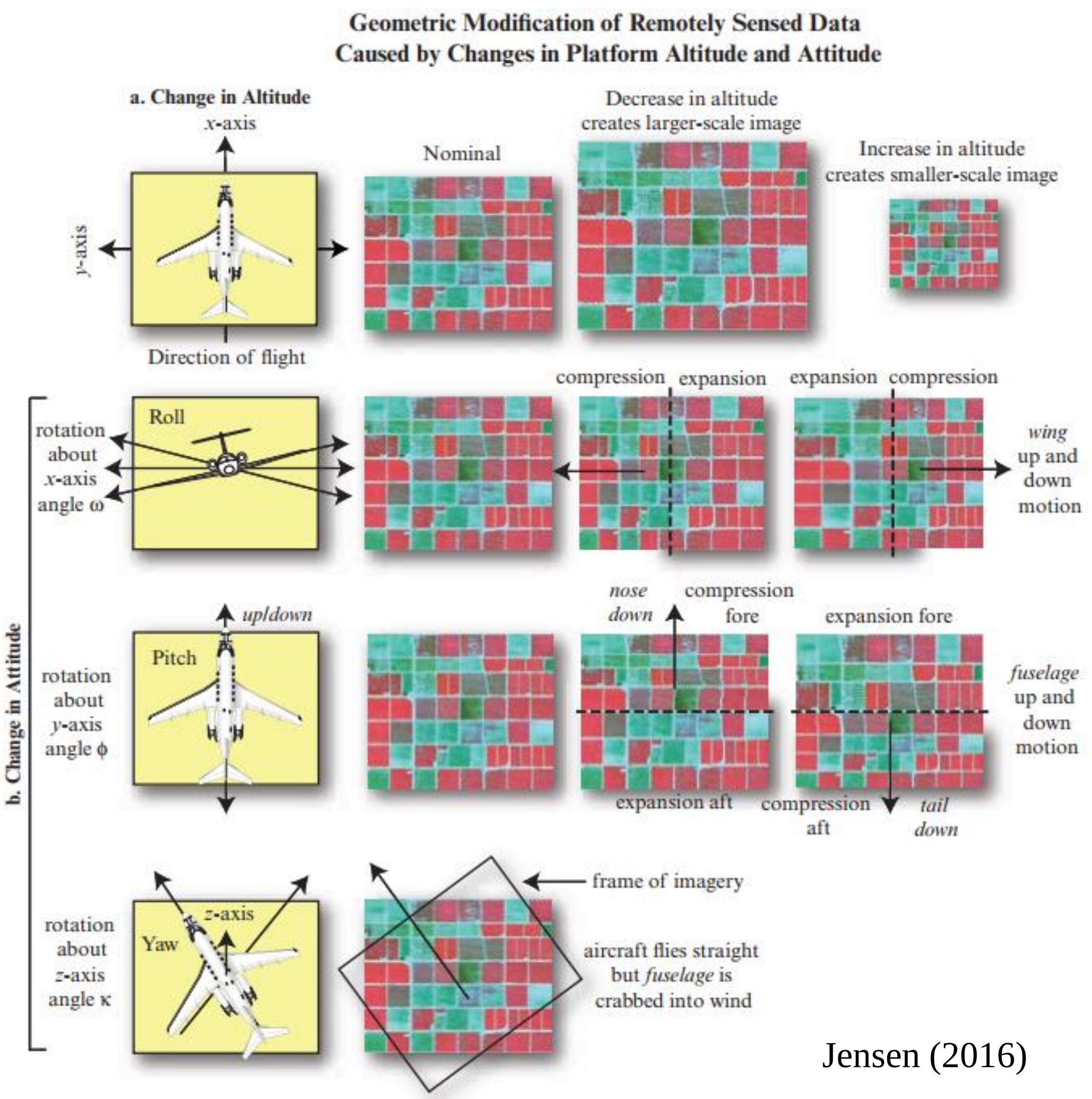


Systematic vs. Random distortions

- Random (non-systematic) distortions
  - Corrected statistically by comparing with ground control points (**non-parametric**)
  - Often done by end-users

Geographic distortions

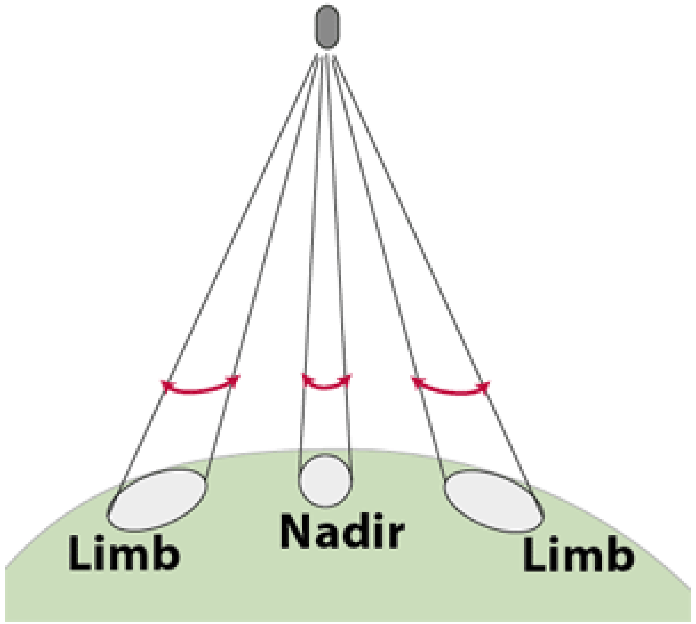
- Geometric distortions in imagery may be due to a variety of factors including one or more of the following:
    - The perspective of the sensor optics
    - The motion of the scanning system
    - The motion and (in)stability of the platform
    - The platform altitude, attitude, and velocity
    - The terrain relief, and
    - The curvature and rotation of the Earth



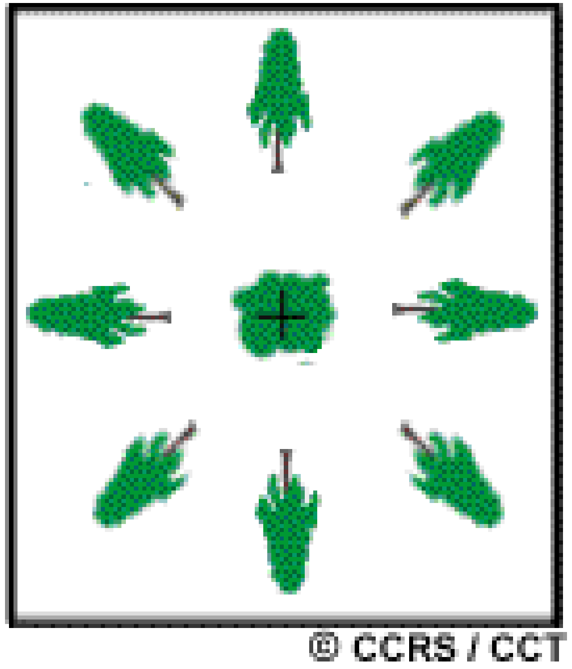
Jensen (2016)

Geographic distortions

- Shape is only preserved at the nadir
- Shape is distorted at the edge of an image



Source: https://www.ssec.wisc.edu/sose/pirs/pirs_m2_footprint.bak



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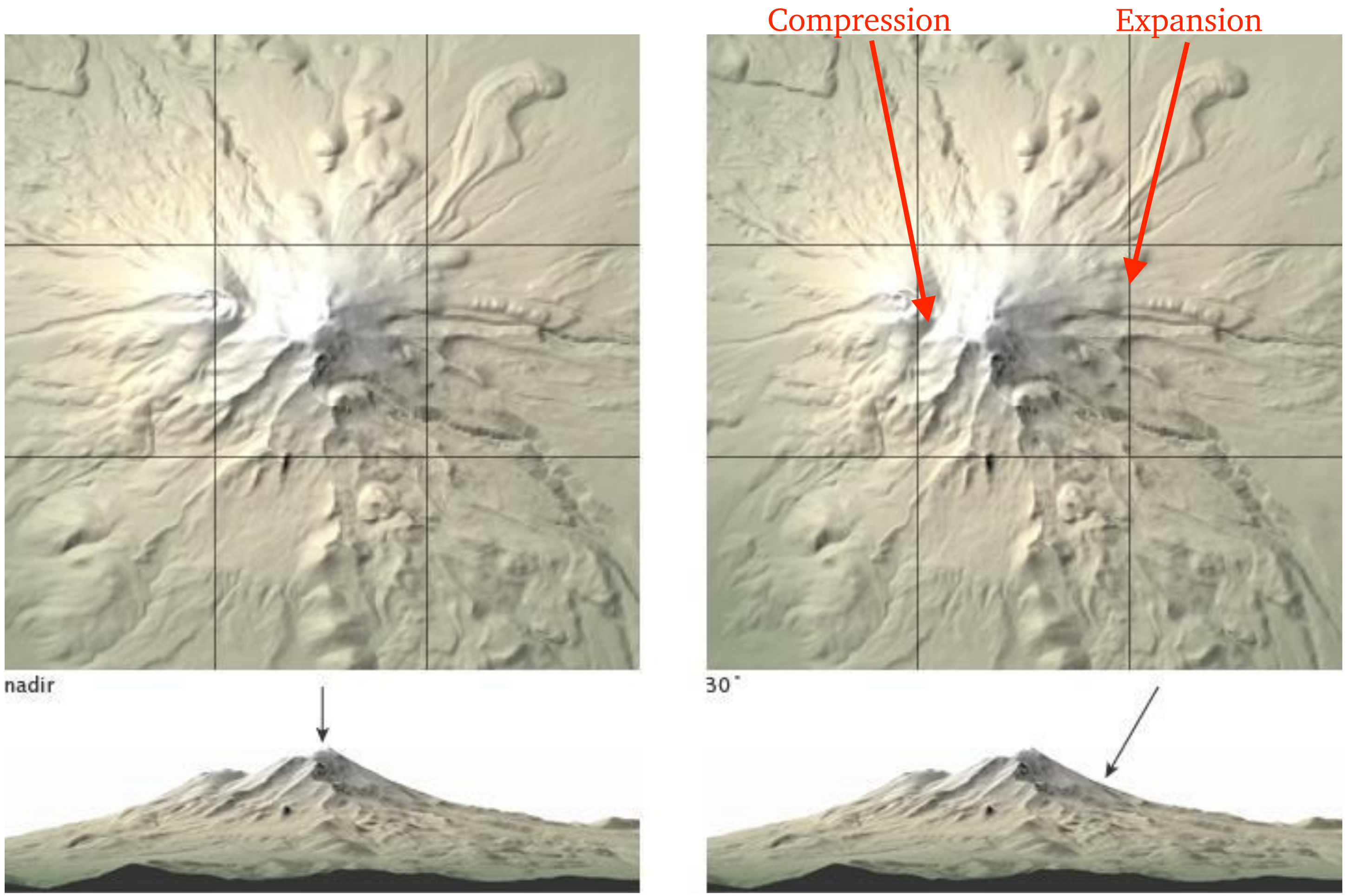
Geographic distortions



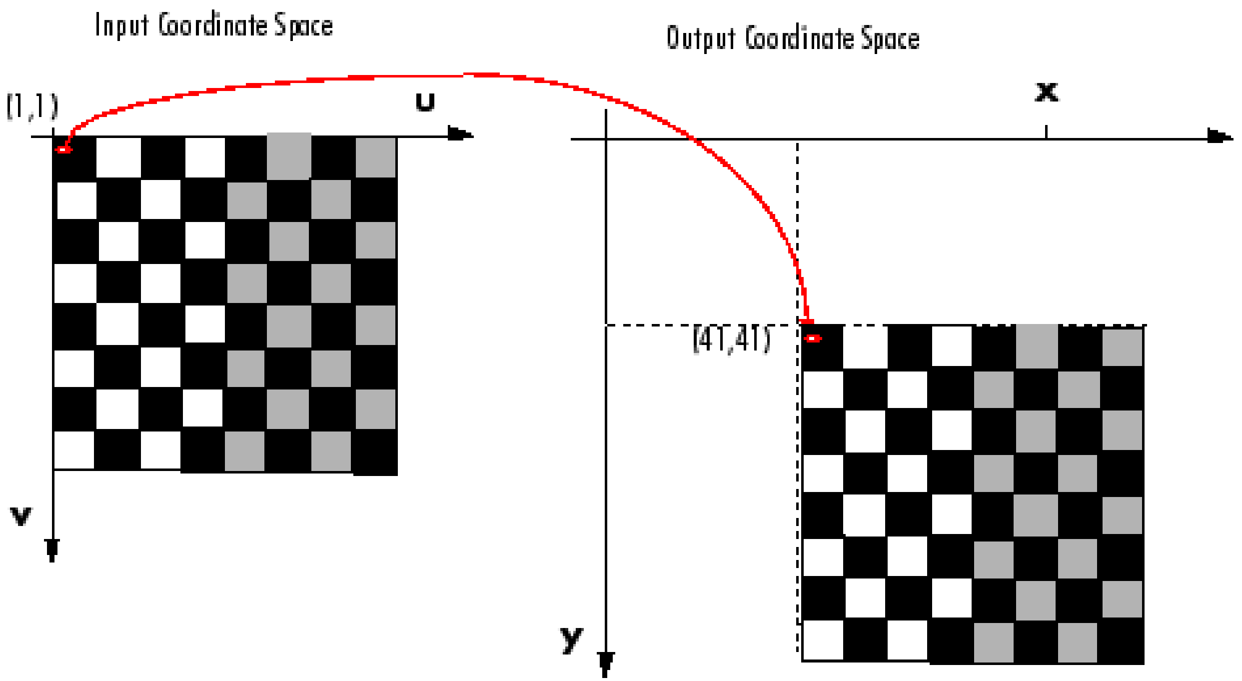
(Source: http://earthobservatory.nasa.gov/Features/GlobalLandSurvey/page3.php)

Types of geometric transformation

- Map-to-map transformation
  - Apply to a **digitized map**
  - Assign a projected coordinate system to the digitized map
  - Convert the map coordinates (e.g., pixel - row 1 & column 3) to projected coordinates
- Image-to-map transformation
  - Apply to **remotely sensed (RS) images**
  - Assign a projected coordinate system to the RS map
  - The original RS image may contain some **distortions**
- **Georeferencing**

Essence of geometric transformation

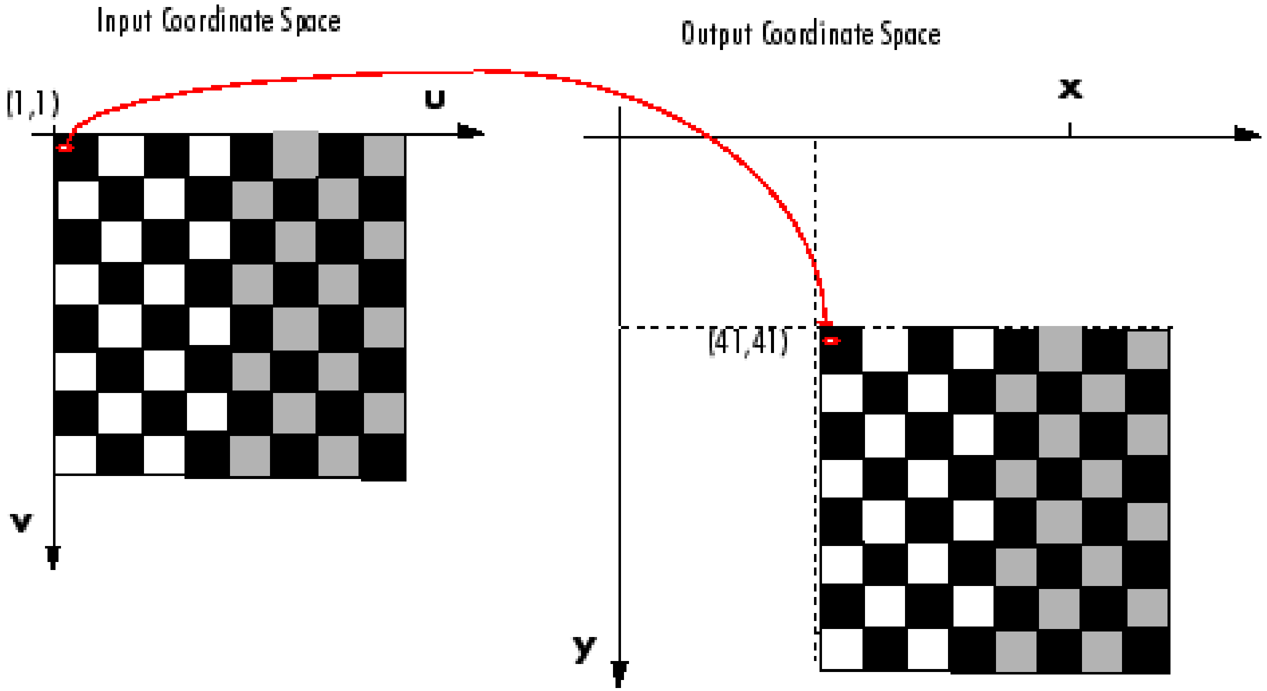
- Essence of geometric transformation
  - Building the mapping relationships between a map coordinate (u,v) before transformation and a map coordinate (x, y) after transformation



$$x = f_1(u,v) \quad y = f_2(u,v)$$

- f_1 and f_2 are the geometric transformation functions with a number of parameters

Essence of geometric transformation



$$x = f_1(u, v) \quad y = f_2(u, v)$$

General steps of geometric transformation

- **Step 1**: Select geometric transformation method (f_1 and f_2 form)
- **Step 2**: Select a number of ground control points (with known x, y, u, v values)
- **Step 3**: Estimate parameters (coefficients) in f_1 and f_2
- **Step4**: Examine the root mean square (RMS) error which is a quantitative measure which determines the quality of geometric transformation
- **Step 5**: Use the estimated coefficients and the transformation equations to compute the new x- and y-coordinates

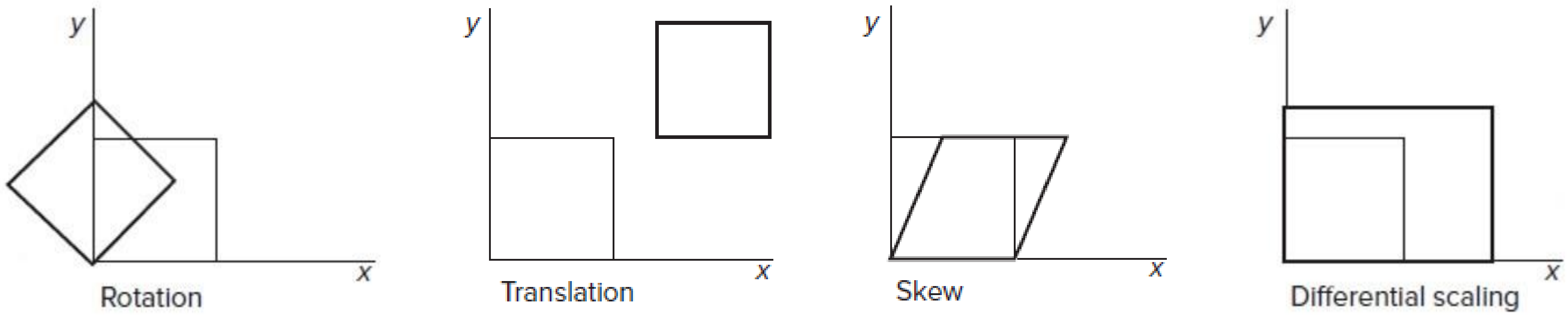
A new map or image with a user defined projected coordinate system

Outline of this lecture

- Background about geometric transformation
- Transformation method
- Control points
- Root Mean Square error
- Resampling

Affine transformation

- The affine transformation allows rotation, translation, skew and differential scaling, while preserving line parallelism. It is also called rubber sheeting



- The affine transformation assumes uniformly distorted input

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Control points

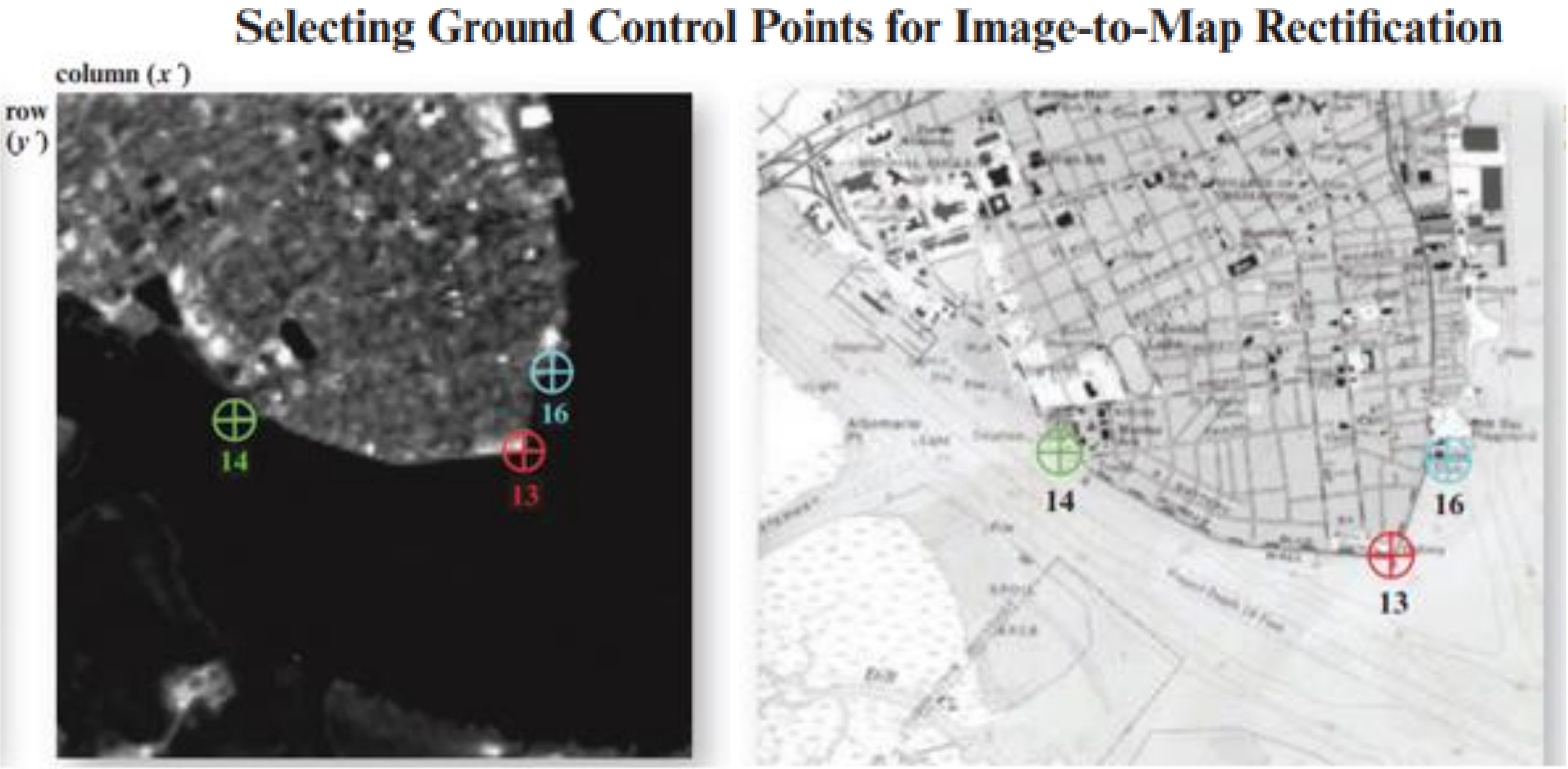
- **Control points** play a key role in determining the accuracy of geometric transformation

Ground control points (GCPs)

Corresponding points

Transformation	Point with input coordinates	Point with output coordinates
Map-to-map	Selected on the source map (usually map intersections)	• Points selected on the reference map • Points with known real-world coordinates
Image-to-map	Selected on the image (features that show up clearly as single distinct pixels, e.g., road intersections, small ponds)	• Points on the reference map • Points with known real-world coordinates

Control points

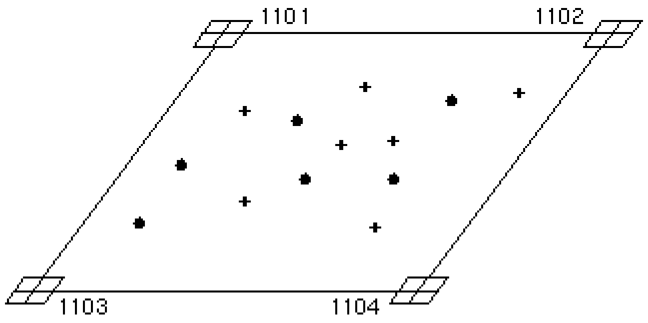


Jensen (2016)

Coordinates on this map are known
We need to build a relationship between the points on this map and the ones on the image

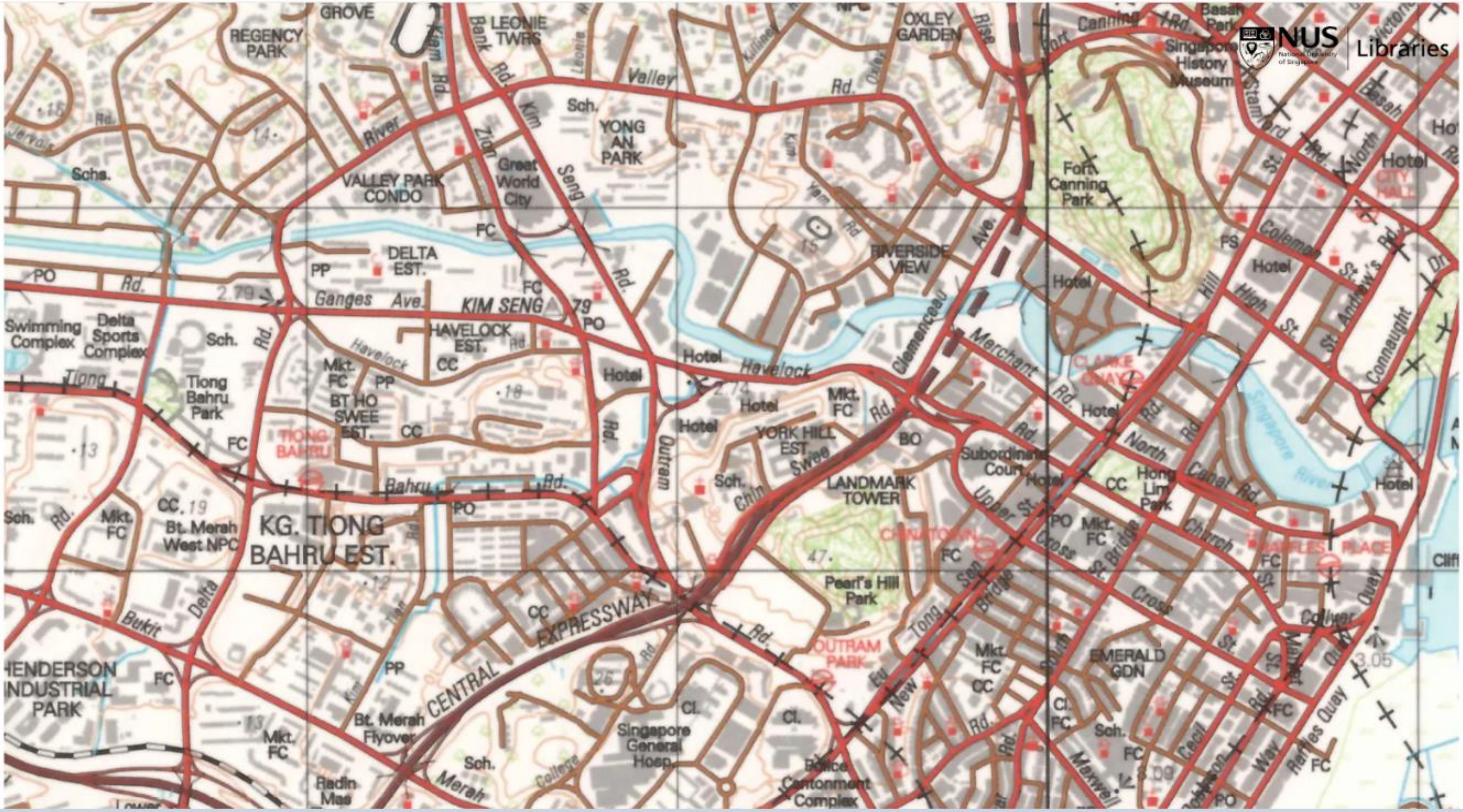
Principles for selecting control points

- Easily identified on both images and maps
  - Maps: Tic points
  - Images: Road intersections, bends of rivers, small prominent features
- Evenly distributed on the images or maps
- Closer to the map features of interest (e.g., GCPs near to NUS as the Area of Interest)



Could you help me select 1 control point?

0



Affine transformation

- The affine transformation is a pair of first-order polynomial equations:

$$x = Au + Bv + C$$
$$y = Du + Ev + F$$

Output coordinates (x, y)

Input coordinates (u, v)

A, B, C, D, E, F are the transformation coefficients

- 3 GCPS
- The number of equations needed should be the same as the number of coefficients.

Affine transformation

- **Second-order** polynomial equations:

$$x = a_0 + a_1 u + a_2 v + a_3 uv + a_4 u^2 + a_5 v^2$$
$$y = b_0 + b_1 u + b_2 v + b_3 uv + b_4 u^2 + b_5 v^2$$

a_0-a_5, b_0-b_5 *are the transformation coefficients*

Control points

Non-linear transformations

Order of Transformation	Minimum GCPs required
1	3
2	6
3	10
4	15
5	21
6	28
7	36
8	45
9	55
10	66

- Often, more than the minimum number of GCPs are used to:
  - Enhance the quality of the geometric transformation

Outline of this lecture

- Background about geometric transformation
- Transformation method
- Control points
- Root Mean Square error
- Resampling

Root Mean Square error

- Root Mean Square (RMS) error is a quantitative measure to determine:
  - The quality of the geometric transformation
  - The goodness of the control points

RMS error measures the deviation between the actual (true) and estimated locations of the control points

Root Mean Square error

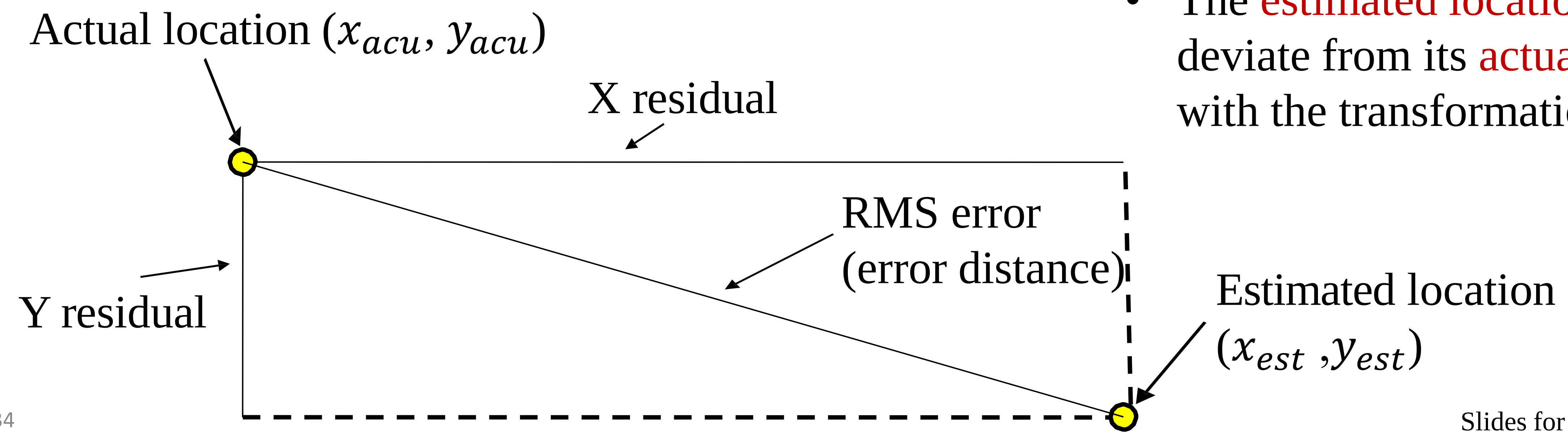
- What are the actual (true) and estimated locations?

$$x = Au + Bv + C$$
$$y = Du + Ev + F$$

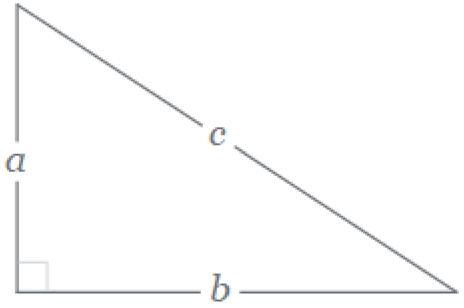
Output coordinates

Input coordinates

- The estimated location can deviate from its actual location with the transformation equation



Root Mean Square error



$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$

The **output RMS error** of the control point $= \sqrt{(x_{acu} - x_{est})^2 + (y_{acu} - y_{est})^2}$

X residual: $(x_{acu} - x_{est})$ Y residual: $(y_{acu} - y_{est})$

If there are 3 control points:

The **total RMS error**

$$= \sqrt{\frac{(x_{acu,1} - x_{est,1})^2 + (y_{acu,1} - y_{est,1})^2 + (x_{acu,2} - x_{est,2})^2 + (y_{acu,2} - y_{est,2})^2 + (x_{acu,3} - x_{est,3})^2 + (y_{acu,3} - y_{est,3})^2}{3}}$$

Root Mean Square error

- The total RMS error:

$$\text{Output} \quad \sqrt{\left[\sum_{I=1}^{n}(x_{acu,i} - x_{est,i})^2 + \sum_{I=1}^{n}(y_{acu,i} - y_{est,i})^2\right]/n}$$

X residual — Y residual

- RMS error can only be computed when the number of GCPs are more than the minimum number required

Root Mean Square error

GCP table

Visible	ID	Source X	Source Y	Dest. X	Dest. Y	dX (map units)	dY (map units)	Residual (map units)
✓	0	4.28644	-3.0161	575672	5.23321e+06	-0.43333	3.95975	3.98339
✓	1	19.7803	-2.96331	585131	5.23334e+06	0.433816	-3.96419	3.98786
✓	2	19.9067	-25.7238	585331	5.21945e+06	-0.432868	3.95553	3.97915
✓	3	4.37903	-25.8023	575850	5.21932e+06	0.432382	-3.95109	3.97468

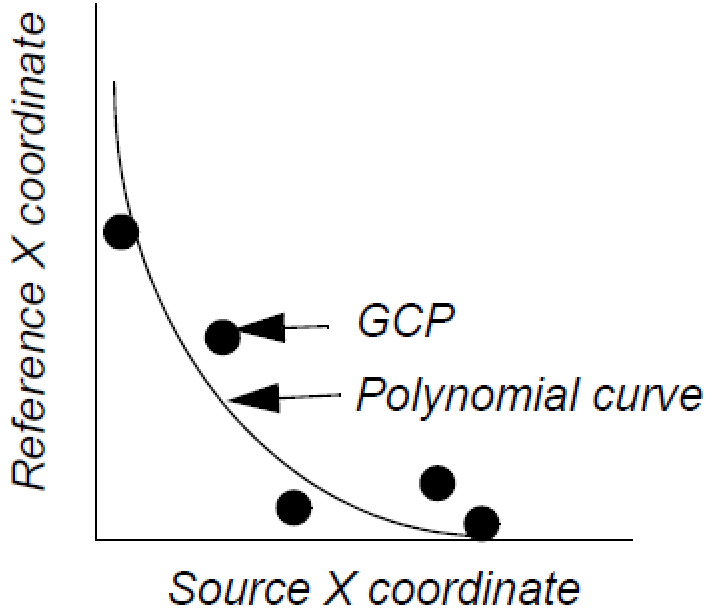
Map: hoytmtn_rect.tif X

1st Order Polynomial (Affine)

	Link	Source X	Source Y	X Map	YMap	Residual X	Residual Y	Residual
✓	1	4.286018	-3.016250	575,672.277100	5,233,212.616300	0.639076	-3.542881	3.600059
✓	2	19.780526	-2.962291	585,131.223200	5,233,341.437100	-0.639717	3.546433	3.603668
✓	3	19.906434	-25.725055	585,331.332700	5,219,450.436000	0.638376	-3.539002	3.596118
✓	4	4.379486	-25.801998	575,850.148000	5,219,321.573000	-0.637735	3.535450	3.592508

Root Mean Square error

- The smaller RMS error is, the better
- But never expect the RMS error to be zero
- To ensure the accuracy of geometric transformation, the RMS error should be within a tolerance value



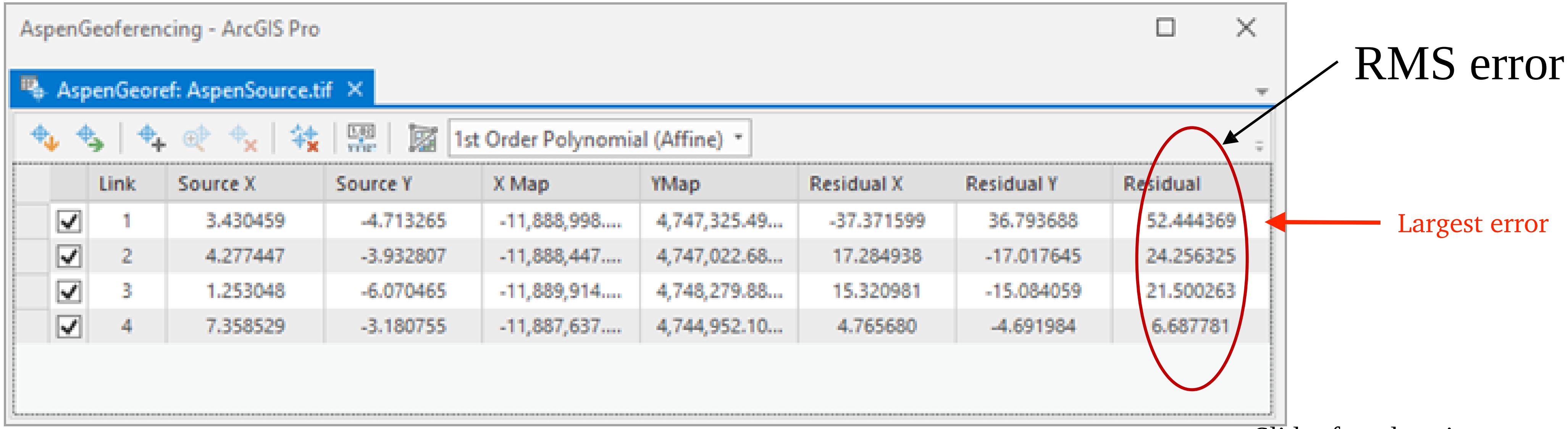
So what tolerance value do you choose?

RMS error tolerance

- The tolerance value is often defined by the data producer
- It can vary by the accuracy and the map scale or by the ground resolution of the input data
  - An RMS error (output) of < 6 meters is acceptable if the input map is a 1:24,000 scale USGS quadrangle map
  - An RMS error (input) of < 1 pixel is probably acceptable for a Landsat Thematic Mapper (TM) scene with a ground resolution of 30 meters

RMS error tolerance

- How to reduce RMS errors?
  1. Choose better control points
  2. Drop the ones with large RMS errors
  3. Choose higher level model
  4. Add more control points



	Link	Source X	Source Y	X Map	YMap	Residual X	Residual Y	Residual
☑	1	3.430459	-4.713265	-11,888,998....	4,747,325.49...	-37.371599	36.793688	52.444369
☑	2	4.277447	-3.932807	-11,888,447....	4,747,022.68...	17.284938	-17.017645	24.256325
☑	3	1.253048	-6.070465	-11,889,914....	4,748,279.88...	15.320981	-15.084059	21.500263
☑	4	7.358529	-3.180755	-11,887,637....	4,744,952.10...	4.765680	-4.691984	6.687781

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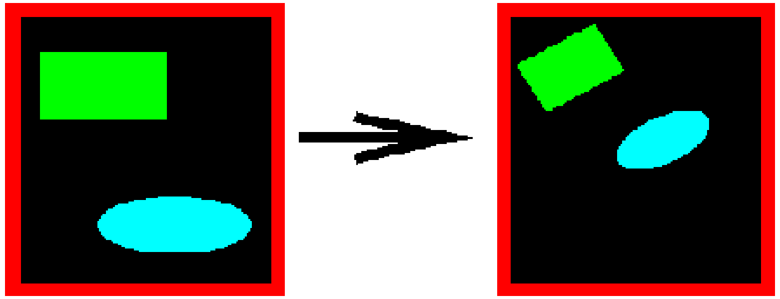
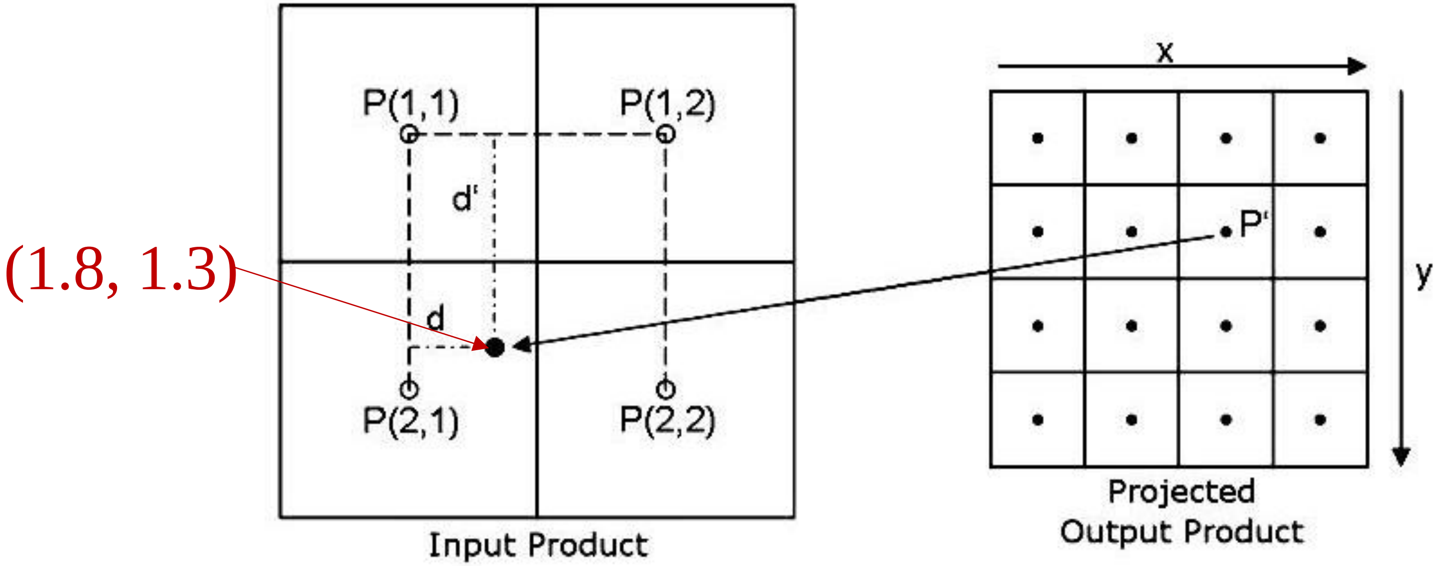
Resampling of pixel values (e.g., elevation value, temperature value)

$$x = Au + Bv + C$$
$$y = Du + Ev + F$$



- The affine transformation equations build the mapping relations between the locations of pixels on the original and new images
- However, the new image has no pixel values (a blank image without pixel values such as temperature values)
- **Resampling** means filling each pixel of the new image with a value or derived value from the original image

Resampling of pixel values



- According to the transformation model, the pixel P' in the output product corresponds to coordinate (1.8, 1.3), which represents the 1.8th row, and the 1.3rd column in the original image.
- So, what should the cell value be for the coordinate (1.8, 1.3)?

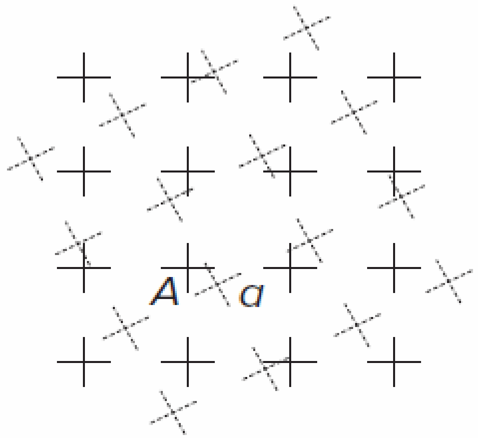
$$x = Au + Bv + C$$
$$y = Du + Ev + F$$

Resampling of pixel values

- Three commonly used resampling methods are:
    1. Nearest neighbor resampling
    2. Bilinear interpolation (distance-weighted)
    3. Cubic convolution (distance-weighted)
- The above three methods are listed in order of increasing complexity and accuracy

Nearest neighbor resampling

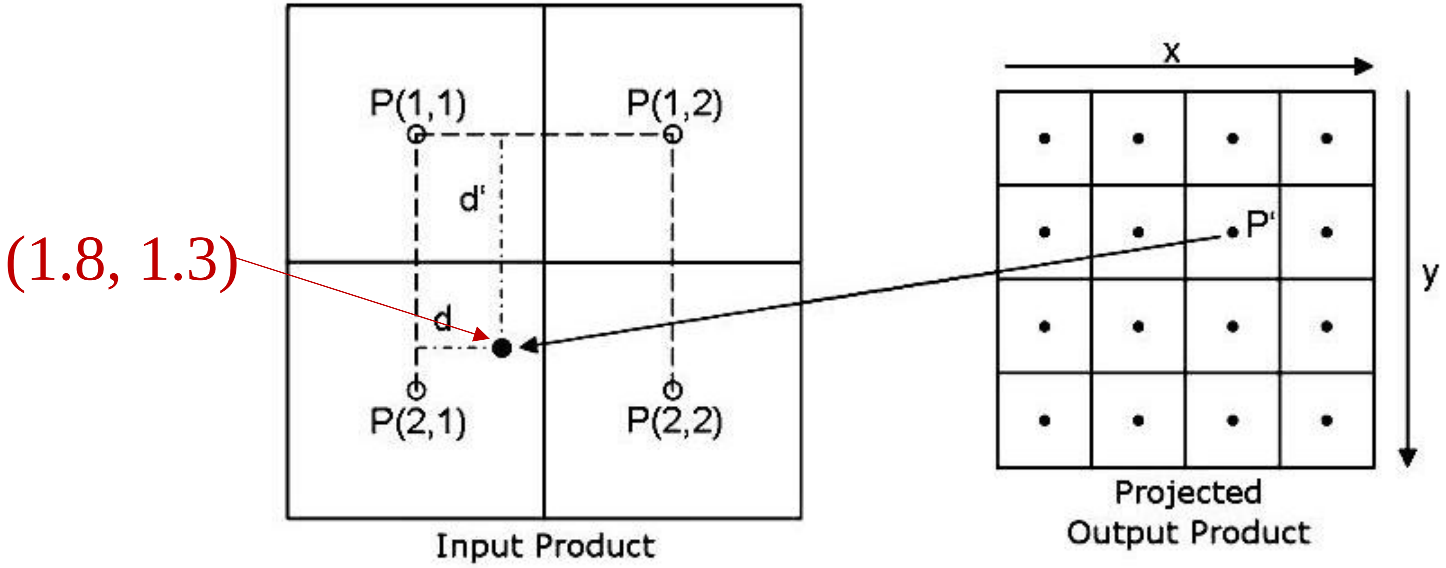
- Step 1: In the new image, the coordinates of each pixel are transformed using the transformation equation to determine its corresponding location in the original image
- Step 2: Find the closest pixel to the corresponding location
- Step 3: Fill the pixel in the new image with the value of the closest pixel mentioned above



There is a pixel in the new image, and its corresponding location in the old image is at *a*. The nearest pixel to *a* is *A*

- **Easy to understand**
- **Quick to compute**
- **Not high in pixel accuracy**

Nearest neighbor resampling

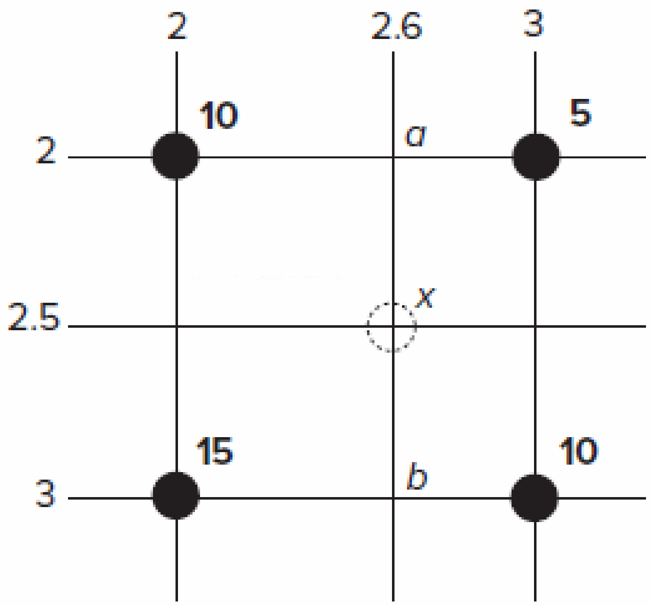


What is the closest pixel to (1.8, 1.3)?

P(2, 1) -- The value of P(2, 1) is assigned to P' (1.8, 1.3)

Bilinear interpolation

- Step 1: In the new image, the coordinates of each pixel are transformed using the transformation equation to determine its corresponding location in the original image (the same)
- Step 2: Find the four nearest pixels to the corresponding location
- Step 3: Fill the pixel in the new image with the value derived from three linear interpolations and the four nearest pixels mentioned above



$$x = Au + Bv + C$$
$$y = Du + Ev + F$$

Bilinear interpolation

Smaller distance means larger weight

- First interpolation:

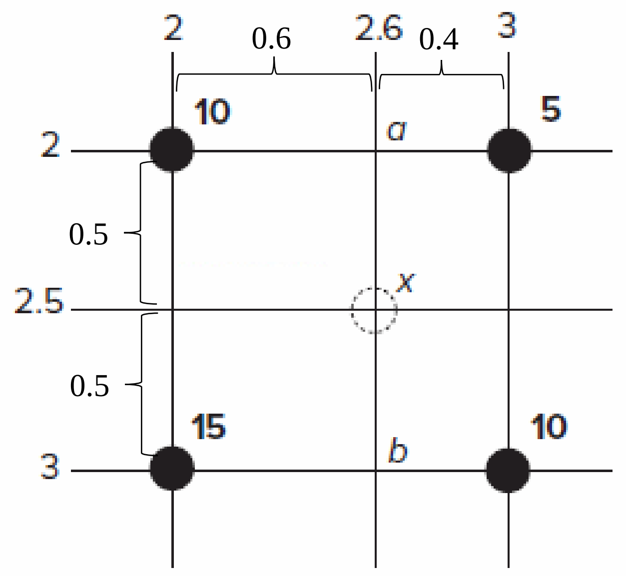
  Pixel value $a = (1 - 0.4) * 5 + (1 - 0.6) * 10 = 7$

- Second interpolation:

  Pixel value $b = (1 - 0.4) * 10 + (1 - 0.6) * 15 = 12$

- Third interpolation:

  Pixel value $x = 0.5 * a + 0.5 * b = 9.5$



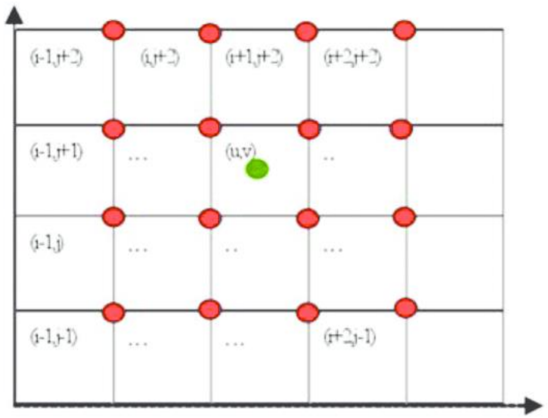
x (2.5, 2.6) corresponds to the location in the old image

Fast to compute

Smoother results

More accurate than nearest neighbor

Cubic convolution



- Find the **16** nearest pixels
- Five cubic polynomial interpolations (i.e., non-linear interpolation) to generate the average pixel value
- Can sharpen the image
- Produce smoother results (remove noise) than bilinear interpolation
- Computationally intensive (takes much longer time to process)

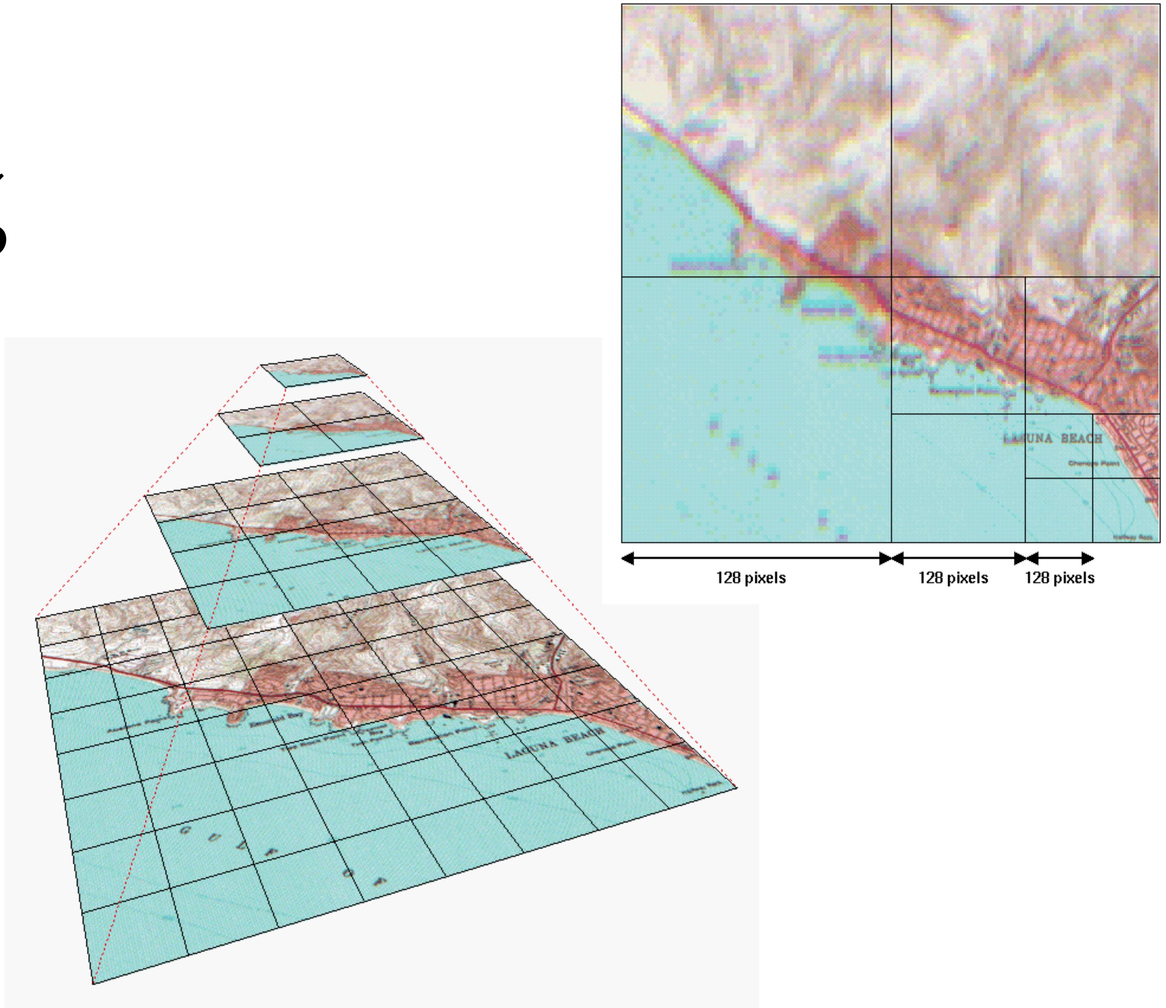
Other uses of resampling

- Resampling is needed whenever there is a change of cell location or cell size between the input raster and the output raster
- Apart from geometric transformation, resampling is also involved in:
  - Projecting a raster from one coordinate system to another

https://source.opennews.org/articles/choosing-right-map-projection/

Other uses of resampling

- Resampling is needed whenever there is a change of cell location or cell size between the input raster and the output raster
- Apart from geometric transformation, resampling is also involved in:
  - Projecting a raster from one coordinate system to another
  - Pyramiding: a common technique for displaying large raster data sets



http://www.ai.sri.com/digital-earth/proposal/3d-representation

Image pyramiding is used in Google map to speed the displaying process of satellite images

Summary

- Background about geometric transformation
  - Why geometric transformation?
  - Geographic distortions
  - Two types of geometric transformation
    - Map-to-map and image-to-map transformation
- Transformation method
  - Affine transformation
- Control points
  - Minimum number required
  - Principles of selecting control points

Summary

- Root Mean Square (RMS) error
  - How to compute RMS error?
  - How to reduce RMS error?
- Resampling
  - What is resampling?
  - Three commonly used resampling methods
    - Nearest neighbor
    - Bilinear interpolation
    - Cubic interpolation
  - Other uses of resampling

THANK YOU